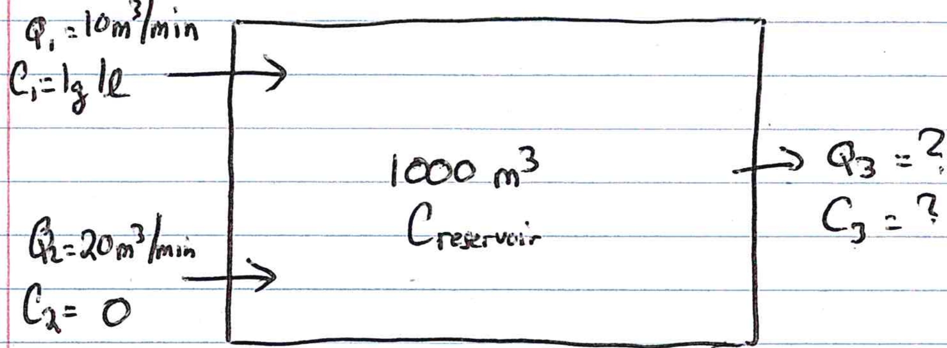
Example



$$\Sigma Q_{in} = \Sigma Q_{out} \Rightarrow Q_3 = Q + Q_2 = 30\ m^3/min$$

Assume $C_3 = C_{reservoir}$ (well mixed)

Conservation of Mass $\Rightarrow \frac{d}{dt}(C_r V) = \dot{m}_{in} - \dot{m}_{out}$

$$= Q_1 C_1 + Q_2 C_2 - Q_3 C_3$$

$$\therefore V \frac{dC_r}{dt} = Q_1 C_1 + Q_2 C_2 - Q_3 C_3$$

$$\frac{dC_r}{dt} = \frac{Q_1 C_1}{V} + \frac{Q_2 C_2}{V} - \frac{Q_3 C_3}{V}$$

$$\frac{dC_r}{dt} = \frac{(10)(1)}{(1000)} + \frac{(20)(0)}{(1000)} - \frac{(30) C_r}{1000}$$

$$\frac{dC_r}{dt} = -0.03 C_r + 0.01$$

Long run $\Rightarrow$ $\frac{dC_r}{dt} = 0$ $\Rightarrow$ $0.01 = 0.03\, C_r$

$\Rightarrow C_r = 1/3\ kg/m^3$

$\rightarrow$ 2 parts clean
1 part dirty

More general $\Rightarrow$ $\frac{dC_r}{dt} = 0.01 - 0.03\, C_r$

~~Guess~~

Educated Guess $\Rightarrow$ Exponential $\left(\frac{dC}{dt} \propto C \right)$

$\Rightarrow$ Starts at 0

$$\Rightarrow C_r = C_{final} \left(1 - e^{-\alpha t}\right) \quad \text{Guess}$$

$$= \frac{1}{3}\left(1 - e^{-0.03t}\right) \quad \text{By substitution}$$

Quiz

$\tau = L^2/D$

$D = 10^{-9}$

$L = 0.1 \Rightarrow \tau = \frac{(0.1)^2}{10^{-9}} = 10^{7}\ s$

$L = 1 \Rightarrow \tau = \frac{1^2}{10^{-9}} = 10^{9}\ s$

$L = 10 \Rightarrow \tau = \frac{10^2}{10^{-9}} = 10^{11}\ s$

$L = 100\ m \Rightarrow \tau = \frac{100^2}{10^{-9}} = 10^{13}\ s$

Sample Problem

$v = 0.5$ m/s

$M = 100\,g = 0.1$ kg

$$\left.\begin{aligned} D_m &= 10^{-9}\ m^2/s \\ D_{mech} &= 2\times10^{-2}\ m^2/s \\ D_{turb} &= 0\ m^2/s \end{aligned}\right\} D = 10^{-9} + 2\times10^{-2} + 0 = 2\times10^{-2}\ m^2/s$$

$$C = \frac{M}{\sqrt{4\pi D t}}\, e^{-\frac{(x-vt)^2}{4Dt}}$$

$$x = 100 \Rightarrow C = \frac{0.1}{\sqrt{8\pi\times10^{-2}\,t}}\, e^{-\frac{(100-0.5t)^2}{8\times10^{-2}\,t}}$$

$$C_{max} \ @ \ t = \frac{x}{v} = \frac{100}{0.5} = 200\,s$$

$$\Rightarrow C_{max} = \frac{0.1}{\sqrt{16\pi}} = 1.4\times10^{-2}\ kg/m$$

$$C_{max} \sim \frac{1}{\sqrt{t}} \rightarrow$$ To fall by a factor of 10 need t 100 times bigger

$$\therefore\ t = 20000\,s \Rightarrow C_{max} = \frac{0.1}{\sqrt{\pi(8\times10^{-2})20000}} = 1.4\times10^{-3}\ kg/m$$

$$x = \cancel{0.5}\ vt = (0.5)(20000) = 10000\ m$$

Breakthrough Curves ~ Temporal Moments

$$\left.\begin{array}{l} M_0 = 5 \ \text{kg s/m} \\ M_1 = 2520 \ \text{kg s}^2/\text{m} \end{array}\right\} \text{from data}$$

$$M = 1 \Rightarrow \quad 5 = \frac{1}{v} \Rightarrow v = 1/5 \ \text{m/s}$$

$$M_1 = 2520 = \frac{2D + vx}{v^3} M$$

$$= \frac{2D + \left(\frac{1}{5}\right)(100)}{(1/5)^3} M$$

$$D = \frac{2520 \, v^3}{2M} - \frac{vx}{2}$$

$$= \frac{(2520)(0.2)^3}{2(1)} - \frac{(0.2)(100)}{2}$$

$$= 8 \times 10^{-2} \ \text{m}^2/\text{s}$$

Sample Problem

$x_1' = 0$

$x_2' = 10$

$$\left.\begin{aligned} C_1 &= \frac{M_1}{(4\pi D t)^{1/2}} e^{-\frac{(x - x_1' - vt)^2}{4Dt}} \\ C_2 &= \frac{M_2}{(4\pi D t)^{1/2}} e^{-\frac{(x - x_2' - vt)^2}{4Dt}} \end{aligned}\right\} C = C_1 + C_2$$

$M_1 = 6$ kg

$M_2 = 4$ kg

$x_1' = 0$ m

$x_2' = 10$ m

$v = 2$ m/s

$D = 0.5$ m^2/s

$x = 100$ m

$$\therefore \quad C = \frac{1}{(4\pi (0.5) t)^{1/2}} \left[6 e^{-\frac{(100 - 2t)^2}{4(0.5)t}} + 4 e^{-\frac{(100 - 10 - 2t)^2}{4(0.5t)}} \right]$$

3 contaminant spills to account for

C_1 $\quad 5 < x < 10$ $\quad\quad$ 3 kg/m

C_2 $\quad -10 < x < -5$ $\quad\quad$ 5 kg/m

C_3 $\quad -3 < x < 3$ $\quad\quad$ 4 kg/m

Recall $C = \frac{C_0}{2}\left[\mathrm{erfc}\left(\frac{x-x_0-vt}{\sqrt{4Dt}}\right) - \mathrm{erfc}\left(\frac{x-x_1-vt}{\sqrt{4Dt}}\right)\right]$

$$C_1 = \frac{3}{2}\left[\mathrm{erfc}\left(\frac{x-10-vt}{\sqrt{4Dt}}\right) - \mathrm{erfc}\left(\frac{x-5-vt}{\sqrt{4Dt}}\right)\right]$$

$$C_2 = \frac{4}{2}\left[\mathrm{erfc}\left(\frac{x+5-vt}{\sqrt{4Dt}}\right) - \mathrm{erfc}\left(\frac{x+10-vt}{\sqrt{4Dt}}\right)\right]$$

$$C_3 = \frac{5}{2}\left[\mathrm{erfc}\left(\frac{x-3-vt}{\sqrt{4Dt}}\right) - \mathrm{erfc}\left(\frac{x+3-vt}{\sqrt{4Dt}}\right)\right]$$

$$C = C_1 + C_2 + C_3$$

See Linear_Sup_Steps.m

for Plot