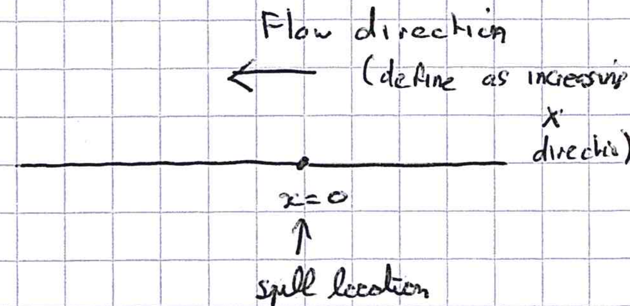
Sample Problem



Information Provided

$M = 1000$ kg

$K = 500$ m/d

$n = 0.3$

depth to water @ $x = -100$ = 1m
" " " @ $x = 200$ = 2 m

$D_{mol} = 1 \times 10^{-9}$ m²/s = 8.64×10^{-5} m²/day

$\alpha = 0.01$ m

① Calculate Darcy velocity

$$|q| = \left|K \frac{\Delta h}{\Delta x}\right| = 500 \frac{(2-1)}{(200-(-100))} = \frac{500}{300} = 5/3 \text{ m day}^{-1}$$

② Calculate Advection velocity

$$v = q/n = \frac{(5/3)}{0.3} = 50/9 \text{ m day}^{-1}$$

$$= 5.55 \text{ m day}^{-1}$$

③ Calculate Dispersion

$$D = D_{mol} + \alpha v$$

$$= (8.64 \times 10^{-5}) + (0.01)(5.55) \text{ m day}^{-1}$$

$$= 5.55 \times 10^{-2} \text{ m day}^{-1}$$ (note D_{mol} is basically negligible)

④ Retardation?

No information ⇒ Assume $R = 1$

⑤ Apply formula $C = \frac{M}{(4\pi D t)^{1/2}} e^{-\frac{(x - x' - vt)^2}{4Dt}}$

case (i) ⇒ 2 m east of spill ⇒ $x = -2$

$$\therefore C = \frac{1000}{(4\pi (5.55\times10^{-2}) t)^{1/2}} e^{-\frac{(-2 - 5.55t)^2}{4(5.55\times10^{-2})t}}$$

t in days

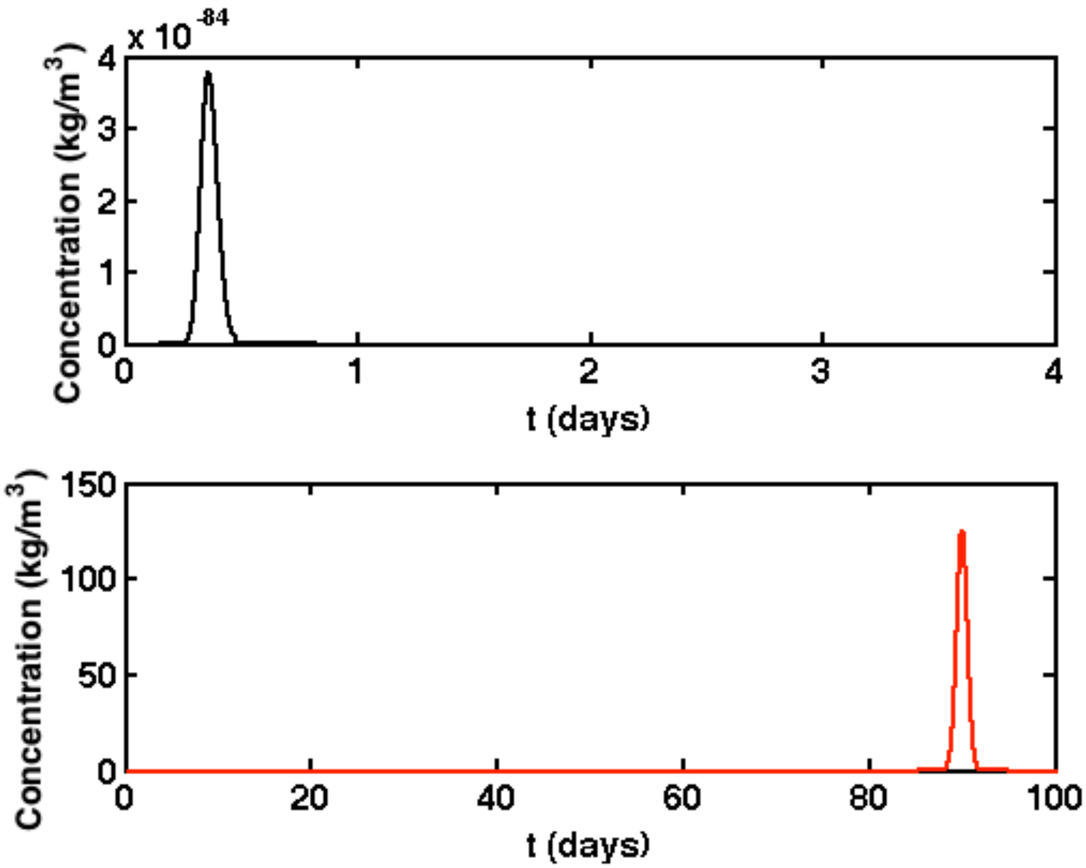
case (ii) ⇒ 500 m west of spill ⇒ $x = 500$

$$\therefore C = \frac{1000}{(4\pi (5.55\times10^{-2}) t)^{1/2}} e^{-\frac{(500 - 5.55t)^2}{4(5.55\times10^{-2})t}}$$

Plot both (see next page)

Note ~ concentrations @ $x = -2$ are tiny!!

↳ Dispersion can only bring so much back against the flow!



Concentrations at the two wells. The top curve is for x=-2 and the bottom one for x=500

Sample Question - Moments

Recall $m_1 = vt$

$m_2 = v^2t^2 + 2Dt$

Use data of m_1 vs t to get $v \rightsquigarrow$ slope

$\Rightarrow v \approx 1$

Harder to get D from m_2

Define $K_{11} = m_2 - m_1^2 = 2Dt$

Use data of K_{11} vs t to get $D \rightsquigarrow \frac{1}{2}$ slope

$$2D \approx \frac{12000}{1000} = 12$$

$$D \approx 6$$

Sample Problem Health Risk

Information Provided

$$\gamma = 0.1 \text{ day}^{-1}$$

$$v = 1 \text{ m/day}$$

$$\alpha = 10^{-3} \text{ m}$$

$$\left.\begin{matrix} v \\ \alpha \end{matrix}\right\} \quad D = \alpha v = 10^{-3} \text{ m}^2 \text{ day}^{-1}$$

$$C_o = 100 \text{ mg/l}$$

$$C = C_o \, e^{-\frac{\left(-v + \sqrt{v^2 + 4\gamma D}\right)}{2D} x}$$

$$= 100 \, e^{-\frac{\left(-1 + \sqrt{1 + 4 \times 10^{-4}}\right)}{2(10^{-3})} x}$$

$$\boxed{C = 100 \, e^{-0.1 x}}$$

Risk Part

$$CPF = 2 \times 10^{-3} \text{ kg day/g}$$

$$EF = 11/12$$

$$ED = 30 \text{ years}$$

$$AT = 70 \text{ years}$$

$$IU = 3 \text{ l day}^{-1}$$

$$BW = 65 \text{ kg}$$

$$Risk = C \quad CPF \quad \frac{IV}{BW} \quad \frac{ED \ EF}{AT}$$

$$= 100 e^{-0.1x} \left(2 \times 10^{-3}\right) \frac{(3)}{(65)} \frac{(30)(4/12)}{(70)}$$

$$= 0.0036 e^{-0.1x}$$

$$= 3.6 \times 10^{-3} e^{-0.1x}$$

(dimensionless

↓

all units cancel in above expression)

$$\therefore \quad x = -10 \ln \left(\frac{Risk}{3.6 \times 10^{-3}} \right)$$

If $Risk = 10^{-6} \Rightarrow x = 81.9 \text{ m}$

$= 10^{-4} \Rightarrow x = 35.8 \text{ m}$