

## Equation Sheet 2

### Reynold Transport Theorem

$$\frac{DM_{\text{sys}}}{Dt} = \frac{\partial M_{\text{cv}}}{\partial t} + \rho_2 A_2 V_2 - \rho_1 A_1 V_1$$

### Chapter 5 – Control Volume Analysis

#### Conservation of Mass

$$\frac{\partial}{\partial t} \int_{\text{cv}} \rho dV + \sum \rho_{\text{out}} A_{\text{out}} V_{\text{out}} - \sum \rho_{\text{in}} A_{\text{in}} V_{\text{in}} = 0$$

#### Conservation of Momentum

$$\frac{\partial}{\partial t} \int_{\text{cv}} V \rho dV + \sum V_{\text{out}} \rho_{\text{out}} A_{\text{out}} V_{\text{out}} - \sum V_{\text{in}} \rho_{\text{in}} A_{\text{in}} V_{\text{in}} = \sum F_{\text{contents CV}}$$

#### Moment of Momentum

$$\sum (\mathbf{r} \times \mathbf{V})_{\text{out}} \rho_{\text{out}} A_{\text{out}} V_{\text{out}} - \sum (\mathbf{r} \times \mathbf{V})_{\text{in}} \rho_{\text{in}} A_{\text{in}} V_{\text{in}} = \sum (\mathbf{r} \times \mathbf{F})_{\text{contents of the control volume}}$$

#### Torque

$$T_{\text{shaft}} = -\dot{m}_{\text{in}}(\pm r_{\text{in}} V_{\theta \text{in}}) + \dot{m}_{\text{out}}(\pm r_{\text{out}} V_{\theta \text{out}})$$

#### Power

$$\dot{W}_{\text{shaft}} = -\dot{m}_{\text{in}}(\pm U_{\text{in}} V_{\theta \text{in}}) + \dot{m}_{\text{out}}(\pm U_{\text{out}} V_{\theta \text{out}})$$

#### Work Per Unit Mass

$$w_{\text{shaft}} = -(\pm U_{\text{in}} V_{\theta \text{in}}) + (\pm U_{\text{out}} V_{\theta \text{out}})$$

## Energy Equation

$$\frac{p_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + gz_{\text{out}} = \frac{p_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + gz_{\text{in}} + w_{\text{shaft net in}} - \text{loss}$$

## In terms of head

$$\frac{p_{\text{out}}}{\gamma} + \frac{V_{\text{out}}^2}{2g} + z_{\text{out}} = \frac{p_{\text{in}}}{\gamma} + \frac{V_{\text{in}}^2}{2g} + z_{\text{in}} + h_s - h_L$$

$$h_s = w_{\text{shaft net in}}/g = \frac{\dot{W}_{\text{shaft net in}}}{\dot{m}g} = \frac{\dot{W}_{\text{shaft net in}}}{\gamma Q}$$

## Chapter 6

### Volumetric Rate of Dilatation

$$\frac{1}{\delta V} \frac{d(\delta V)}{dt} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \nabla \cdot \mathbf{V}$$

### Vorticity

$$\omega_z = \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

$$\omega_x = \frac{1}{2} \left( \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right)$$

$$\omega_y = \frac{1}{2} \left( \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right)$$

Strain rate

$$\dot{\gamma} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

Conservation of Mass

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

Cylindrical

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial(r\rho v_r)}{\partial r} + \frac{1}{r} \frac{\partial(\rho v_\theta)}{\partial \theta} + \frac{\partial(\rho v_z)}{\partial z} = 0$$

Streamfunctions

$$u = \frac{\partial \psi}{\partial y} \quad v = -\frac{\partial \psi}{\partial x}$$

$$v_r = \frac{1}{r} \frac{\partial \psi}{\partial \theta} \quad v_\theta = -\frac{\partial \psi}{\partial r}$$

## Navier Stokes Equations

$$\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial x} + \rho g_x + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

$$\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = -\frac{\partial p}{\partial y} + \rho g_y + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right)$$

$$\rho \left( \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial p}{\partial z} + \rho g_z + \mu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right)$$

## Navier Stokes Equations (Cylindrical)

$$\begin{aligned} \rho \left( \frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z} \right) \\ = -\frac{\partial p}{\partial r} + \rho g_r + \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_r}{\partial r} \right) - \frac{v_r}{r^2} + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2} \right] \end{aligned}$$

$$\begin{aligned} \rho \left( \frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z} \right) \\ = -\frac{1}{r} \frac{\partial p}{\partial \theta} + \rho g_\theta + \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_\theta}{\partial r} \right) - \frac{v_\theta}{r^2} + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2} \right] \end{aligned}$$

$$\begin{aligned} \rho \left( \frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \right) \\ = -\frac{\partial p}{\partial z} + \rho g_z + \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2} \right] \end{aligned}$$