Chapter 5
Finite Control Volume Analysis

CE30460 - Fluid Mechanics
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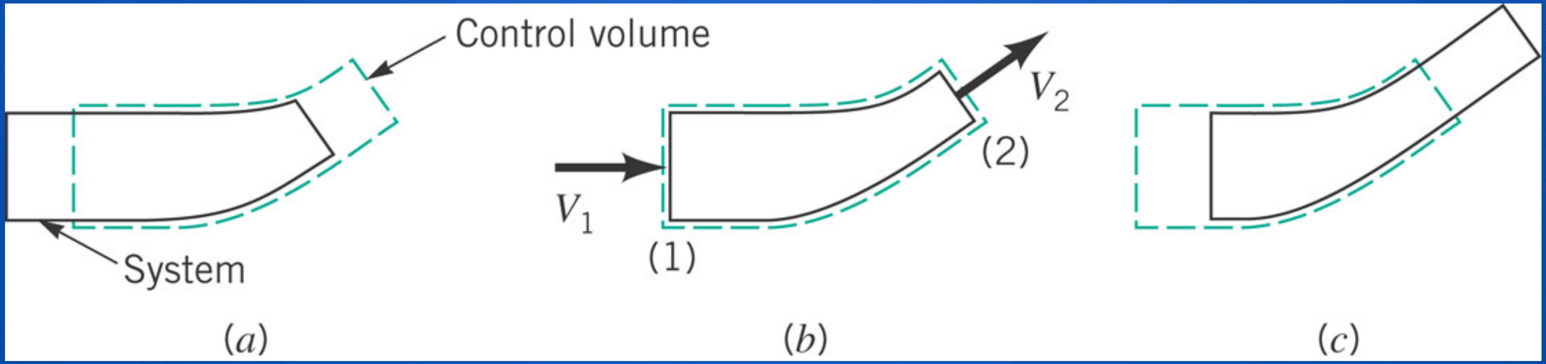
Objectives of this Chapter

- Learn how to select an appropriate control volume
- Understand and Apply the Continuity Equation
- Calculate forces and torques associated with fluid flows using momentum equations
- Apply energy equations to pipe and pump systems
- Apply Kinetic Energy Coefficient

Recall System and Control Volume

- Recall: A system is defined as a collection of unchanging contents
  - What does this mean for the rate of change of system mass?



Recall Control Volume (CV)

$$\frac{DM_{\text{sys}}}{Dt} = 0$$

- Recall Reynolds Transport Theorem (end of last chapter)

$$\frac{DM_{\text{sys}}}{Dt} = \frac{\partial M_{\text{cv}}}{\partial t} + \rho_2 A_2 V_2 - \rho_1 A_1 V_1$$

- Let's look at control volumes on video

Conservation of Mass

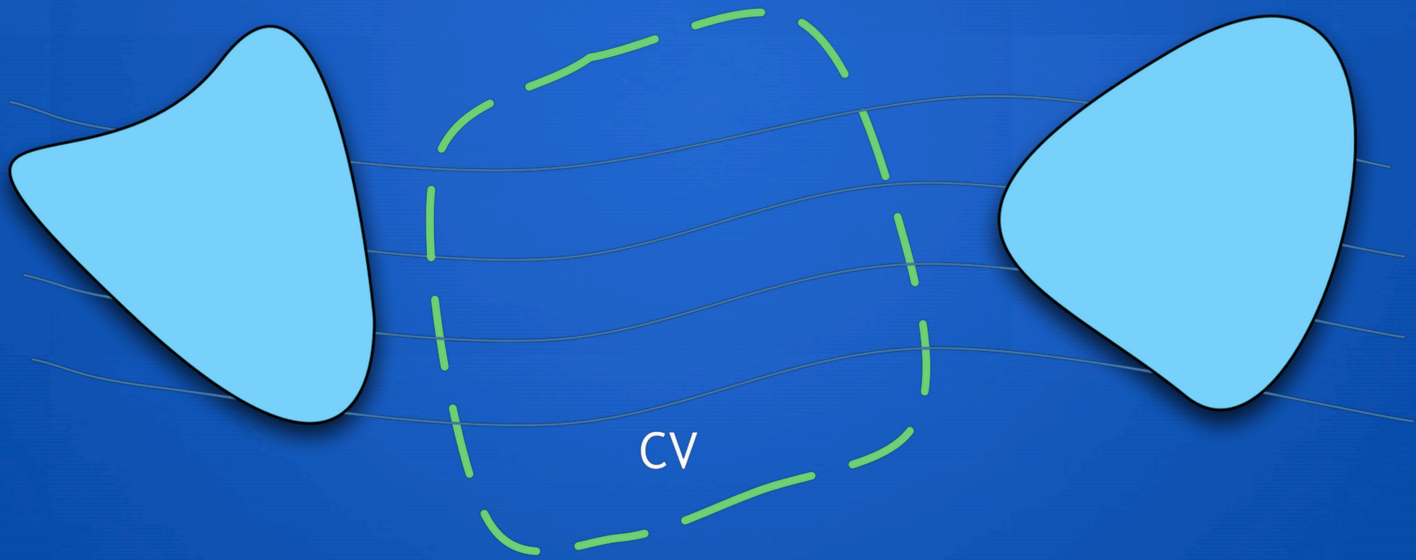
- Combining what we know about the system and the Reynolds Transport Theorem we can write down a equation for conservation of mass, often called 'The Continuity Equation'

$$\frac{\partial}{\partial t}\int_{\mathrm{cv}} \rho \, d\forall + \sum \rho_{\mathrm{out}} A_{\mathrm{out}} V_{\mathrm{out}} - \sum \rho_{\mathrm{in}} A_{\mathrm{in}} V_{\mathrm{in}} = 0$$

- All it is saying is that the total amount of mass in the CV and how that changes depends on how much flows in and how much flows out …

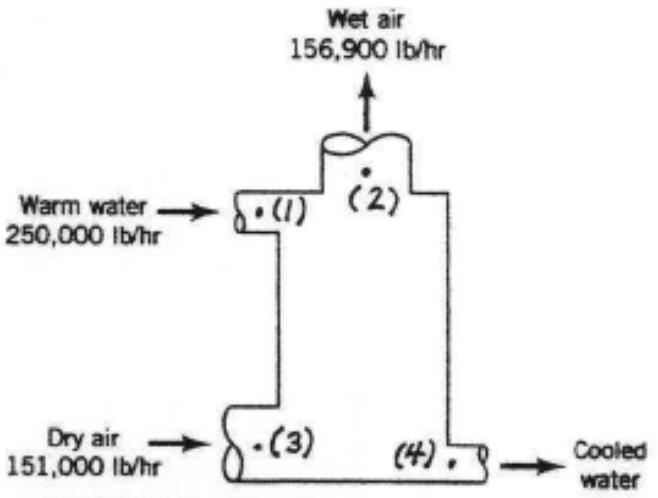
Fixed Non Deforming CV

- Examples



Sample Problem 1

5.11 An evaporative cooling tower (see Fig. P5.11) is used to cool water from 110 to 80°F. Water enters the tower at a rate of 250,000 lb/hr. Dry air (no water vapor) flows into the tower at a rate of 151,000 lb/hr. If the rate of wet air flow out of the tower is 156,900 lb/hr, determine the rate of water evaporation in lb/hr and the rate of cooled water flow in lb/hr.



■ FIGURE P5.11

Sample Problem 2

5.15 An appropriate turbulent pipe flow velocity profile is

$$\mathbf{V} = u_c\left(\frac{R-r}{R}\right)^{1/n}\hat{\mathbf{i}}$$

where u_c = centerline velocity, r = local radius, R = pipe radius, and $\hat{\mathbf{i}}$ = unit vector along pipe centerline. Determine the ratio of average velocity, $\bar{u}$, to centerline velocity, u_c, for (a) $n = 5$, (b) $n = 6$, (c) $n = 7$, (d) $n = 8$, (e) $n = 9$, (f) $n = 10$.

Sample Problem 3

5.17 Two rivers merge to form a larger river as shown in Fig. P5.17. At a location downstream from the junction (before the two streams completely merge), the nonuniform velocity profile is as shown and the depth is 6 ft. Determine the value of V.

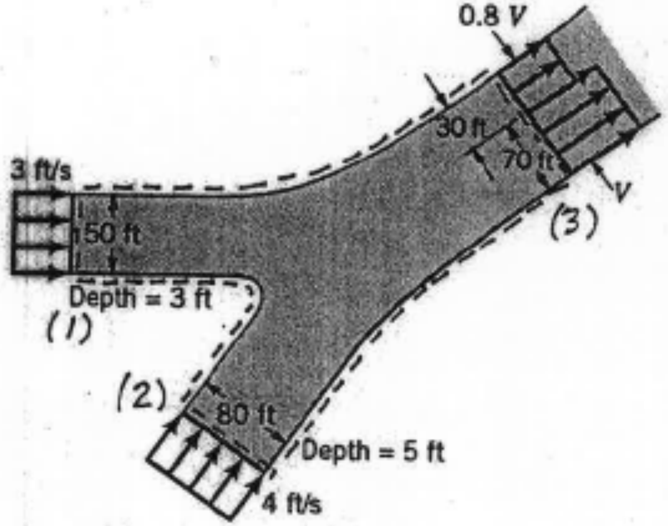


FIGURE P5.17

Sample Problem 4

- Consider a rectangular tank (2mx2m) of height 2m with a hole in the bottom of the tank of size (5cmx5cm) initially filled with water. Water flows through the hole
- Calculate the height of the water level in the tank as it evolves in time
- Assume the coefficient of contraction for the hole is equal to 0.6

Conservation of Mass

- Videos and Pictures

Numbers 867, 882, 884, 885, 886, 889

Multimedia Fluid Mechanics (G.M. Homsy et al), Cambridge University Press

Moving CV

- Example:
  - Bubbles rising: http://www.youtube.com/watch?v=dC55J2TJJYs

Conservation of Momentum

- Newton's Second Law
  - ΣF=ma
  - Or better said :
    - Time rate of change of momentum of the system=sum of external forces acting on the system

$$\frac{D}{Dt}\int_{\text{sys}} \mathbf{V}\rho \, d\forall = \sum \mathbf{F}_{\text{sys}}$$

    - Again, we will apply the Reynolds Transport Theorem (write it out yourselves)

Conservation of Momentum

- General Case

$$\frac{\partial}{\partial t}\int_{CV} V\rho d\forall + \sum V_{out}\rho_{out}A_{out}V_{out} - \sum V_{in}\rho_{in}A_{in}V_{in} = \sum F_{contentsCV}$$

- Steady Flow

$$\sum \mathbf{V}_{\text{out}}\rho_{\text{out}}A_{\text{out}}V_{\text{out}} - \sum \mathbf{V}_{\text{in}}\rho_{\text{in}}A_{\text{in}}V_{\text{in}} = \sum \mathbf{F}_{\substack{\text{contents of the}\\ \text{control volume}}}$$

- Linear Momentum Equation

Relevant Examples

- Fire Hose
  - http://www.youtube.com/watch?v=R8PQTRovFaY&feature=related
  - http://www.break.com/index/firemen-lift-car-with-hose-water.html
- Cambridge Video : 924

Sample Problem 1

5.23 Determine the anchoring force required to hold in place the conical nozzle attached to the end of the laboratory sink faucet shown in Fig. P5.23 when the water flowrate is 10 gal/min. The nozzle weight is 0.2 lb. The nozzle inlet and exit inside diameters are 0.6 and 0.2 in., respectively. The nozzle axis is vertical and the axial distance between sections (1) and (2) is 1.2 in. The pressure at section (1) is 68 psi.

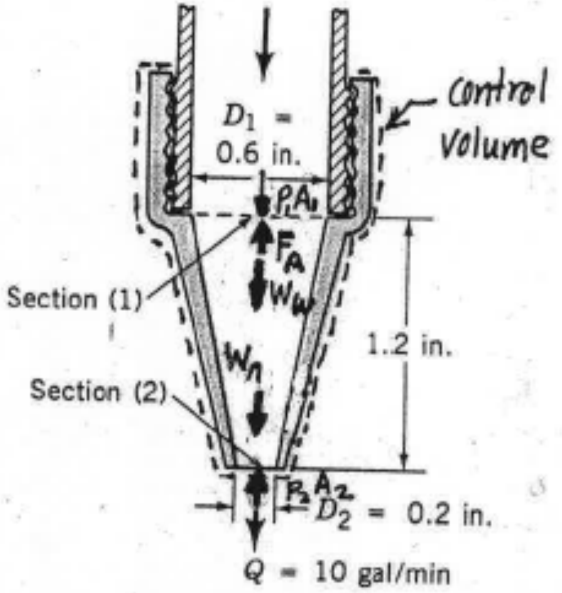
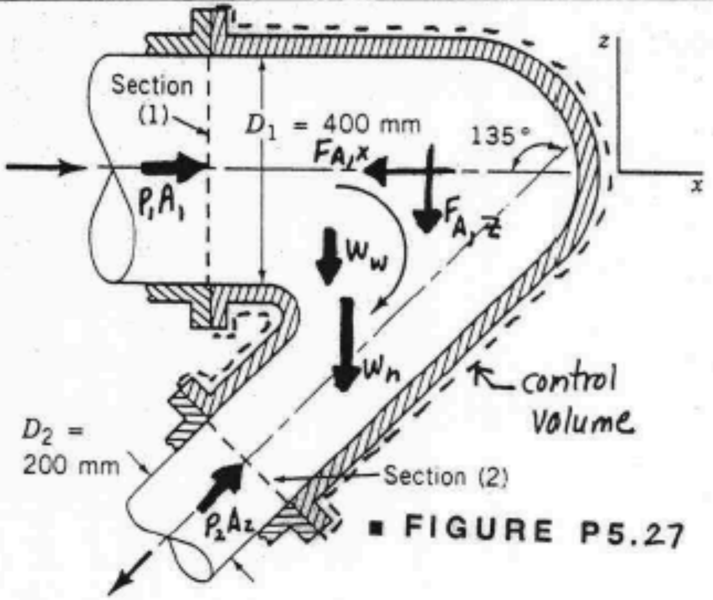


FIGURE P5.23

Sample Problem 2

5.27

5.27 A converging elbow (see Fig. P5.27) turns water through an angle of 135° in a vertical plane. The flow cross section diameter is 400 mm at the elbow inlet, section (1), and 200 mm at the elbow outlet, section (2). The elbow flow passage volume is 0.2 m^3 between sections (1) and (2). The water volume flowrate is 0.4 m^3/s and the elbow inlet and outlet pressures are 150 kPa and 90 kPa. The elbow mass is 12 kg. Calculate the horizontal (x direction) and vertical (z direction) anchoring forces required to hold the elbow in place.

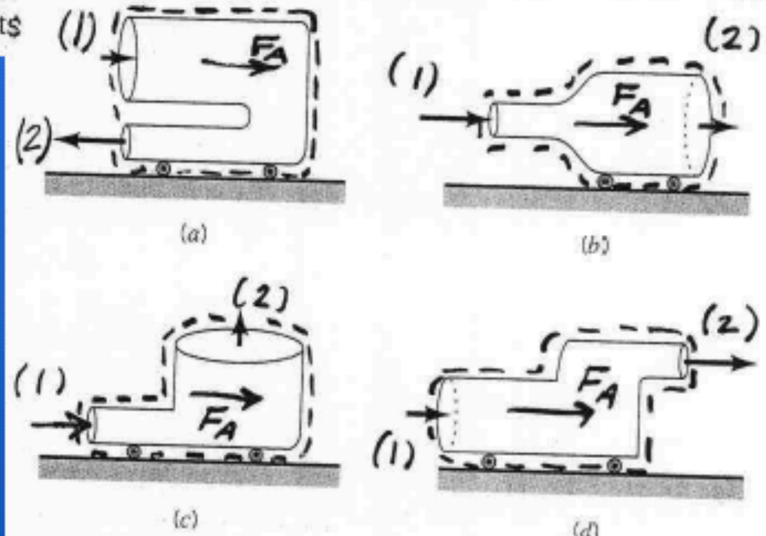


■ FIGURE P5.27

Sample Problem 3

5.32

5.32 The four devices shown in Fig. P5.32 rest on frictionless wheels, are restricted to move in the x direction only and are initially held stationary. The pressure at the inlets and outlets of each is atmospheric, and the flow is incompressible. The contents of each device is not known. When released, which devices will move to the right and which to the left? Explain.

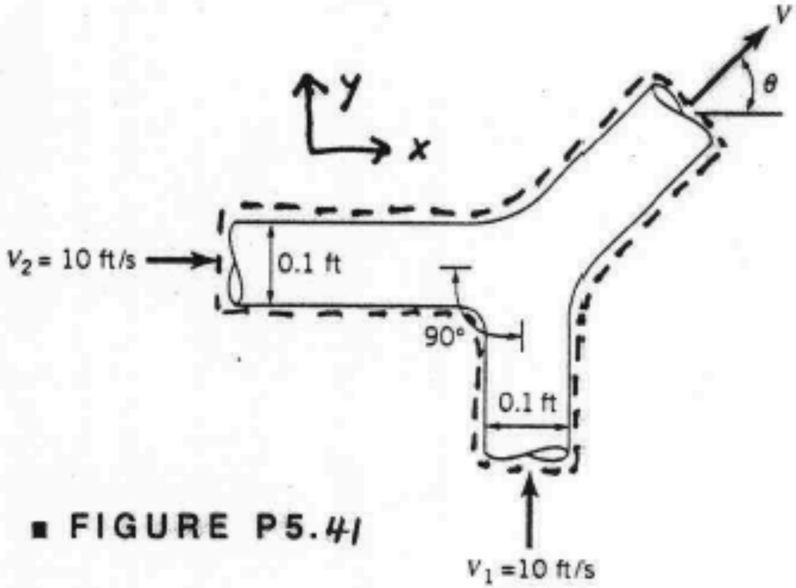


A few comments on linear momentum applications

- Linear Momentum is directional (3 components)
- If a control surface is selected perpendicular to flow entering or leaving surface force is due to pressure
- May need to account for atmospheric pressure
- Sign of forces (direction) is very important
- On external forces (internal forces cancel out – equal and opposite reactions)

Sample Problem 4

5.41 Two water jets of equal size and speed strike each other as shown in Fig. P5.41. Determine the speed, V, and direction, θ, of the resulting combined jet. Gravity is negligible.

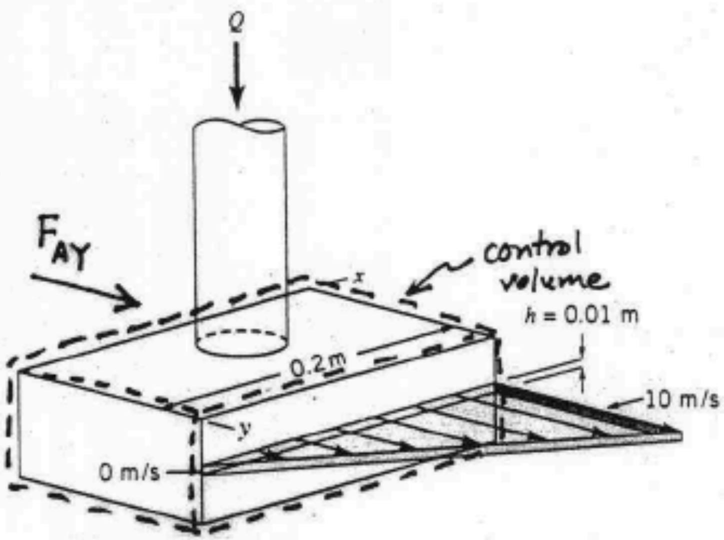


■ FIGURE P5.41

Sample Problem 5

5.49

5.49 A sheet of water of uniform thickness (h = 0.01 m) flows from the device shown in Fig. P5.49. The water enters vertically through the inlet pipe and exits horizontally with a speed that varies linearly from 0 to 10 m/s along the 0.2-m length of the slit. Determine the y component of the anchoring force necessary to hold this device stationary.



■ FIGURE P5.49

Moment of Momentum

- In many application torque (moment of a force with respect to an axis) is important
- Take a the moment of the linear momentum equation for a system

$$\frac{D}{Dt}\int_{\text{sys}} \mathbf{V}\rho \, d\forall = \sum \mathbf{F}_{\text{sys}}$$

$$\frac{D}{Dt}\int_{\text{sys}} (\mathbf{r} \times \mathbf{V})\rho \, d\forall = \sum (\mathbf{r} \times \mathbf{F})_{\text{sys}}$$

Apply Reynolds Transport Theorem

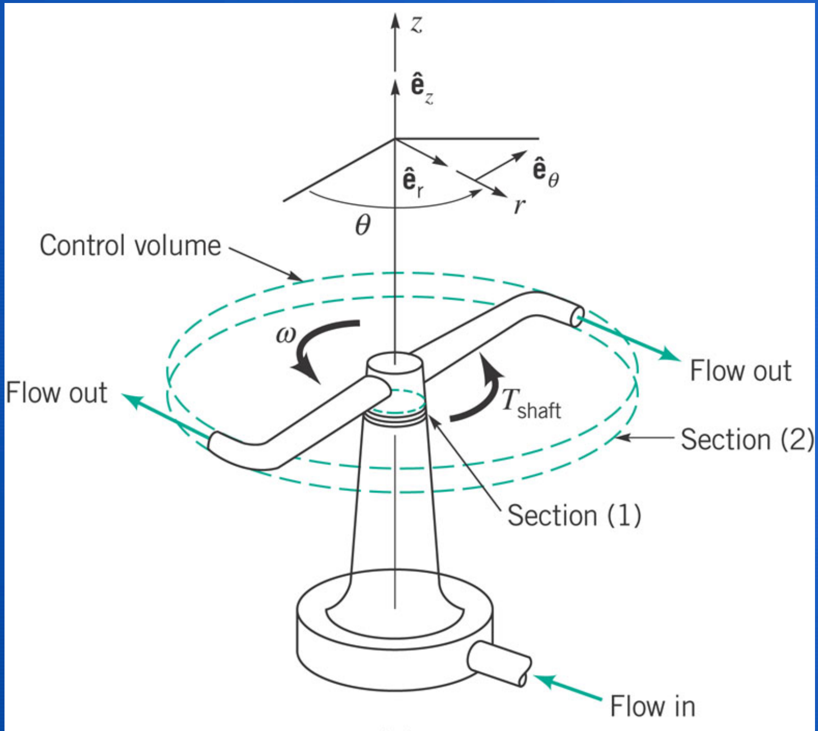
Let's focus on steady problems

- Moment of Momentum Equation for steady flows through a fixed, nondeforming control volume with uniform properties across inlets and outlets with velocity normal of inlets and outlets (more general form available in book Appendix D)

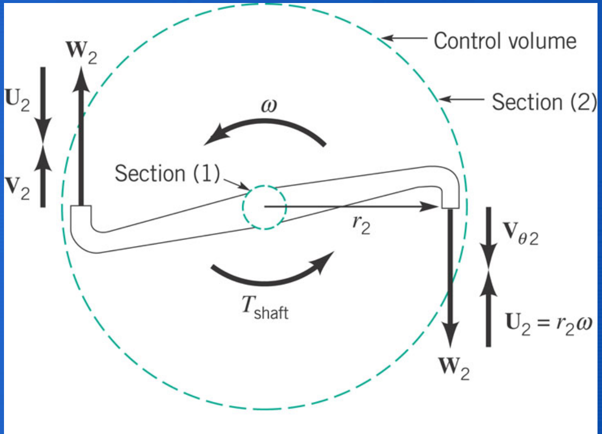
$$\sum(\mathbf{r} \times \mathbf{V})_{\text{out}}\rho_{\text{out}}A_{\text{out}}V_{\text{out}} - \sum(\mathbf{r} \times \mathbf{V})_{\text{in}}\rho_{\text{in}}A_{\text{in}}V_{\text{in}} = \sum(\mathbf{r} \times \mathbf{F})_{\substack{\text{contents of the}\\ \text{control volume}}}$$

- Rotating Machinery

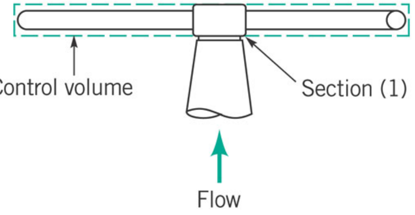
Application (from textbook)



(a)



(b)



(c)

Moment of Momentum Formulas

Torque

$$T_{\text{shaft}} = -\dot{m}_{\text{in}}(\pm r_{\text{in}} V_{\theta\text{in}}) + \dot{m}_{\text{out}}(\pm r_{\text{out}} V_{\theta\text{out}})$$

Power

$$\dot{W}_{\text{shaft}} = -\dot{m}_{\text{in}}(\pm U_{\text{in}} V_{\theta\text{in}}) + \dot{m}_{\text{out}}(\pm U_{\text{out}} V_{\theta\text{out}})$$

Work per Unit Mass

$$w_{\text{shaft}} = -(\pm U_{\text{in}} V_{\theta\text{in}}) + (\pm U_{\text{out}} V_{\theta\text{out}})$$

Sample Problem 1

5.53 Water enters a rotating lawn sprinkler through its base at the steady rate of 16 gal/min as shown in Fig. P5.53. The exit cross section area of each of the two nozzles is 0.04 in.^{2} and the flow leaving each nozzle is tangential. The radius from the axis of rotation to the centerline of each nozzle is 8 in. **(a)** Determine the resisting torque required to hold the sprinkler head stationary. **(b)** Determine the resisting torque associated with the sprinkler rotating with a constant speed of 500 rev/min. **(c)** Determine the angular velocity of the sprinkler if no resisting torque is applied.

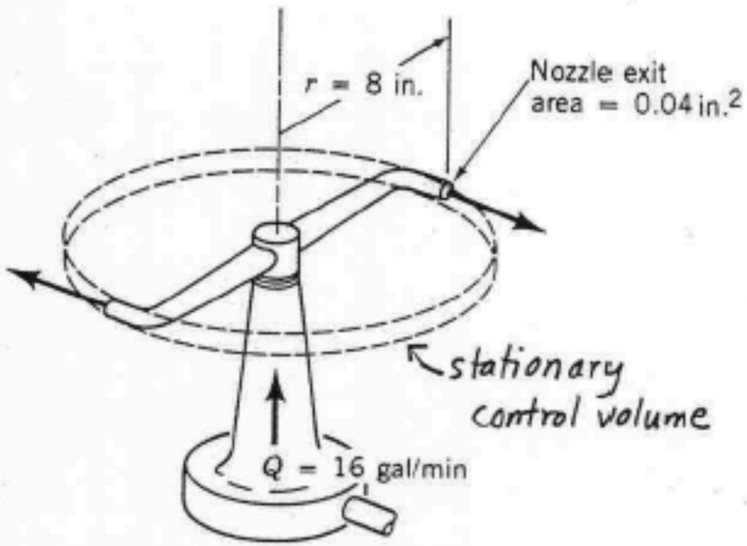
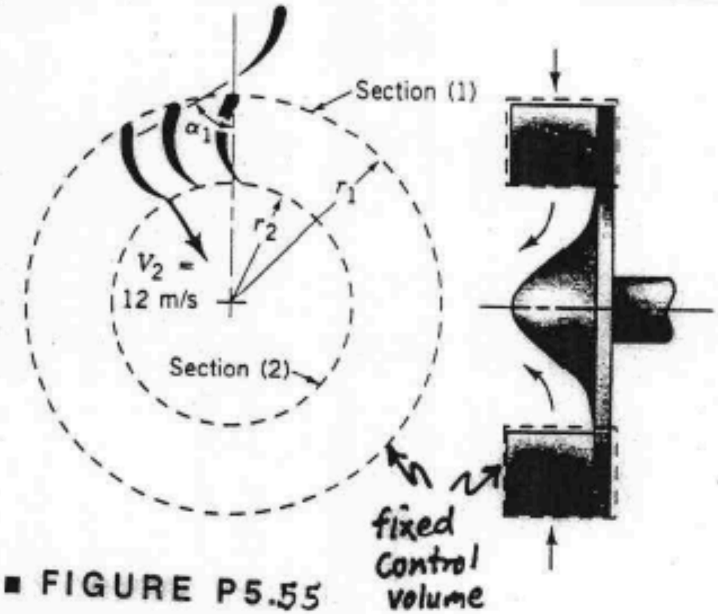


FIGURE P5.53

Sample Problem 2

5.55

5.55 An inward flow radial turbine (see Fig. P5.55) involves a nozzle angle, α_1, of 60° and an inlet rotor tip speed, U_1, of 6 m/s. The ratio of rotor inlet to outlet diameters is 2.0. The absolute velocity leaving the rotor at section (2) is radial with a magnitude of 12 m/s. Determine the energy transfer per unit of mass of fluid flowing through this turbine if the fluid is (a) air or (b) water.



■ FIGURE P5.55

Conservation of Energy
First Law of Thermodynamics

- Same principles as for all conservation laws

Time rate of change of total energy stored

=

Net time rate of energy addition by heat transfer

+

Net time rate of energy addition by work transfer

- We go through the same process transferring system to control volume by Reynolds Transport Theorem

Mathematically Speaking

- First Law of Theromodynamics

$$\frac{D}{Dt}\int_{\text{sys}} e\rho \, d\forall = \left(\sum \dot{Q}_{\text{in}} - \sum \dot{Q}_{\text{out}}\right)_{\text{sys}} + \left(\sum \dot{W}_{\text{in}} - \sum \dot{W}_{\text{out}}\right)_{\text{sys}}$$

- A few definitions
  - Adiabatic – heat transfer rate is zero
  - Power – rate of work transfer $\dot{W}$

Power – comes in various forms

- For a rotating shaft

$$\dot{W}_{\text{shaft}} = T_{\text{shaft}}\omega$$

- For a normal stress (Force x Velocity)

$$\dot{W}_{\text{normal stress}} = p_{\text{in}} A_{\text{in}} V_{\text{in}} - p_{\text{out}} A_{\text{out}} V_{\text{out}}$$

For application purposes

$$\frac{\partial}{\partial t}\int_{\text{cv}} e\rho\, d\forall + \sum\left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz\right)_{\text{out}} \rho_{\text{out}} A_{\text{out}} V_{\text{out}} - \sum\left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz\right)_{\text{in}} \rho_{\text{in}} A_{\text{in}} V_{\text{in}} = \dot{Q}_{\substack{\text{net}\\ \text{in}}} + \dot{W}_{\substack{\text{shaft}\\ \text{net in}}}$$

OR for steady flow….

$$\dot{m}\left[\check{u}_{\text{out}} - \check{u}_{\text{in}} + \left(\frac{p}{\rho}\right)_{\text{out}} - \left(\frac{p}{\rho}\right)_{\text{in}} + \frac{V_{\text{out}}^2 - V_{\text{in}}^2}{2} + g(z_{\text{out}} - z_{\text{in}})\right] = \dot{Q}_{\substack{\text{net}\\ \text{in}}} + \dot{W}_{\substack{\text{shaft}\\ \text{net in}}}$$

Internal energy, enthalpy, kinetic energy, potential energy

Comparison to Bernoulli's Eqn

- For steady, incompressible flow with zero shaft power

$$\frac{p_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + gz_{\text{out}} = \frac{p_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + gz_{\text{in}} - (\check{u}_{\text{out}} - \check{u}_{\text{in}} - q_{\substack{\text{net}\\ \text{in}}})$$

If this is zero – identical
Often treated as a correction
Factor called 'loss'

- Include a source of energy (turbine, pump)

$$\frac{p_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + gz_{\text{out}} = \frac{p_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + gz_{\text{in}} + w_{\substack{\text{shaft}\\ \text{net in}}} - \text{loss}$$

Or in terms of head

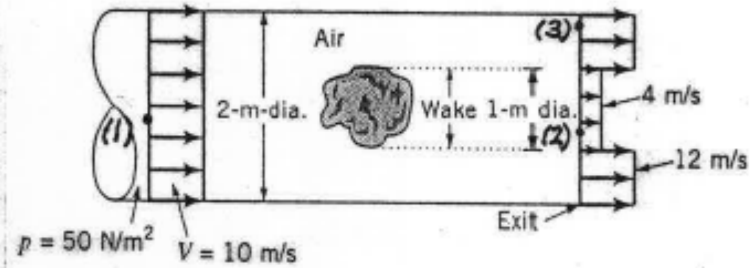
$$\frac{p_{\text{out}}}{\gamma} + \frac{V_{\text{out}}^2}{2g} + z_{\text{out}} = \frac{p_{\text{in}}}{\gamma} + \frac{V_{\text{in}}^2}{2g} + z_{\text{in}} + h_s - h_L$$

$$h_s = w_{\text{shaft net in}}/g = \frac{\dot{W}_{\substack{\text{shaft}\\\text{net in}}}}{\dot{m}g} = \frac{\dot{W}_{\substack{\text{shaft}\\\text{net in}}}}{\gamma Q}$$

Sample Problem 1

5.67

5.67 Air flows past an object in a 2-m-diameter pipe and exits as a free jet as shown in Fig. P5.67. The velocity and pressure upstream are uniform at 10 m/s and 50 N/m^2, respectively. At the pipe exit the velocity is nonuniform as indicated. The shear stress along the pipe wall is negligible. **(a)** Determine the head loss associated with a particle as it flows from the uniform velocity upstream of the object to a location in the wake at the exit plane of the pipe. **(b)** Determine the force that the air puts on the object.



■ FIGURE P5.67

Sample Problem 2

5.73

5.73 An incompressible liquid flows steadily along the pipe shown in Fig. P5.73. Determine the direction of flow and the head loss over the 6-m length of pipe.

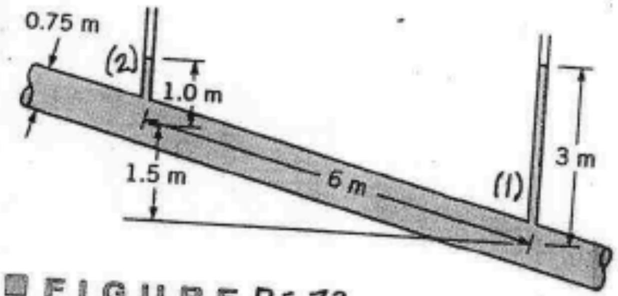


FIGURE P5.73

Application of Energy Equation to Nonuniform Flows

- Modified energy Equation

$$\frac{p_{\text{out}}}{\rho} + \frac{\alpha_{\text{out}}\overline{V}_{\text{out}}^2}{2} + gz_{\text{out}} = \frac{p_{\text{in}}}{\rho} + \frac{\alpha_{\text{in}}\overline{V}_{\text{in}}^2}{2} + gz_{\text{in}} + w_{\substack{\text{shaft}\\ \text{net in}}} - \text{loss}$$

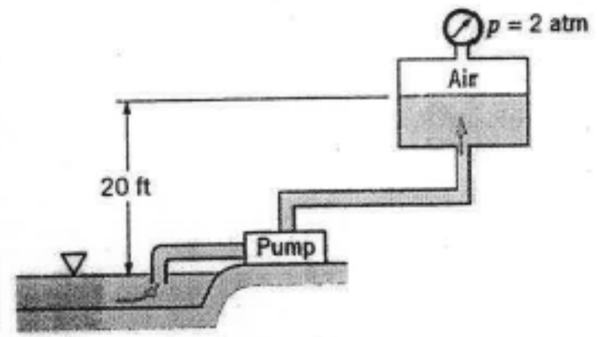
- α – kinetic energy coefficient
- α = 1 for uniform flows,
- α > 1 for nonuniform (tabulated, many practical cases

α ~1) – in this course will be given

Sample Problem 1

5.81

5.81 A pump is to move water from a lake into a large, pressurized tank as shown in Fig. P5.81 at a rate of 1000 gal in 10 min or less. Will a pump that adds 3 hp to the water work for this purpose? Support your answer with appropriate calculations. Repeat the problem if the tank were pressurized to 3, rather than 2, atmospheres.

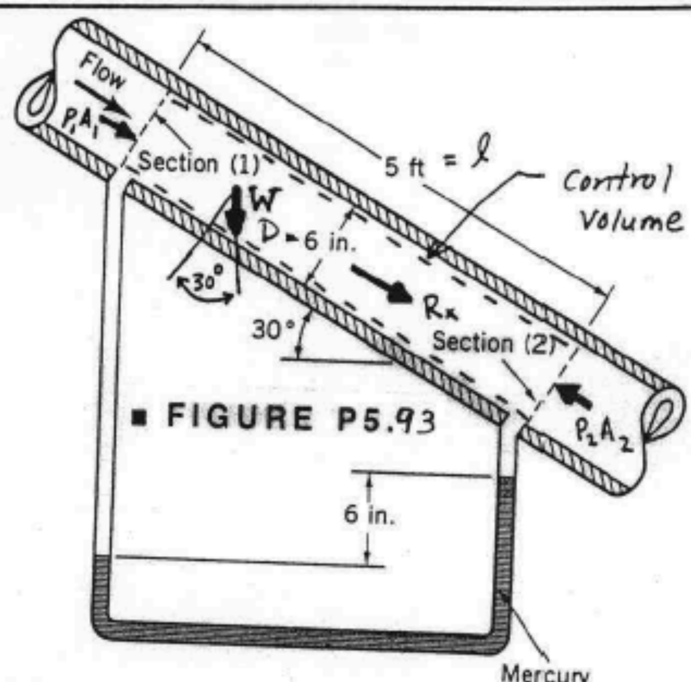


■ FIGURE P5.81

Sample Problem 2

5.93

5.93 Water flows steadily down the inclined pipe as indicated in Fig. P5.93. Determine the following: **(a)** the difference in pressure $p_1 - p_2$, **(b)** the loss between sections (1) and (2), and **(c)** the net axial force exerted by the pipe wall on the flowing water between sections (1) and (2).



■ FIGURE P5.93