Chapter 4
Fluid Kinematics

CE30460 - Fluid Mechanics

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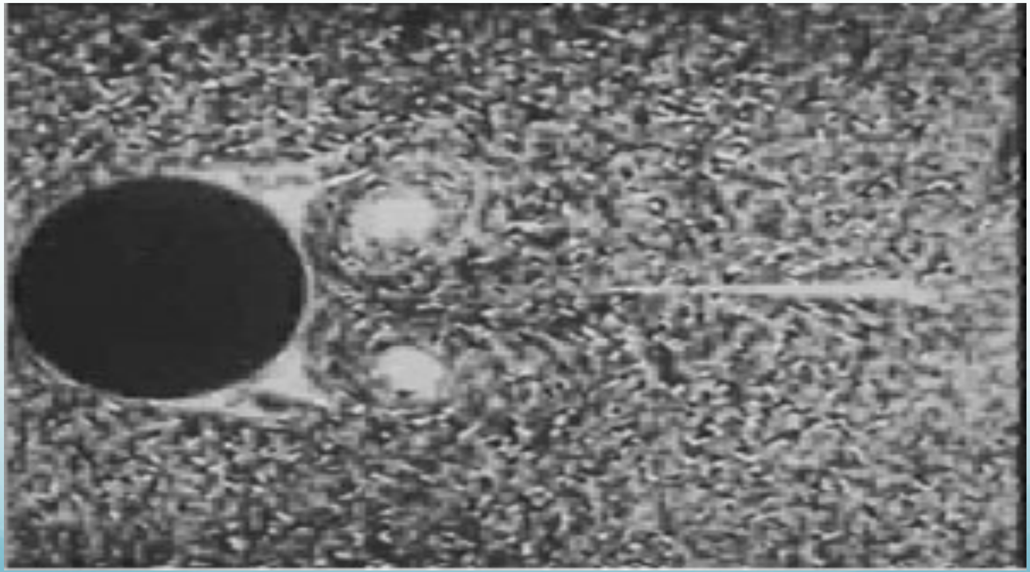
Velocity Field

$$\mathbf{V} = u(x, y, z, t)\hat{\mathbf{i}} + v(x, y, z, t)\hat{\mathbf{j}} + w(x, y, z, t)\hat{\mathbf{k}}$$

- How could you visualize a velocity field in a real fluid?



Streamlines, Steaklines and Pathlines

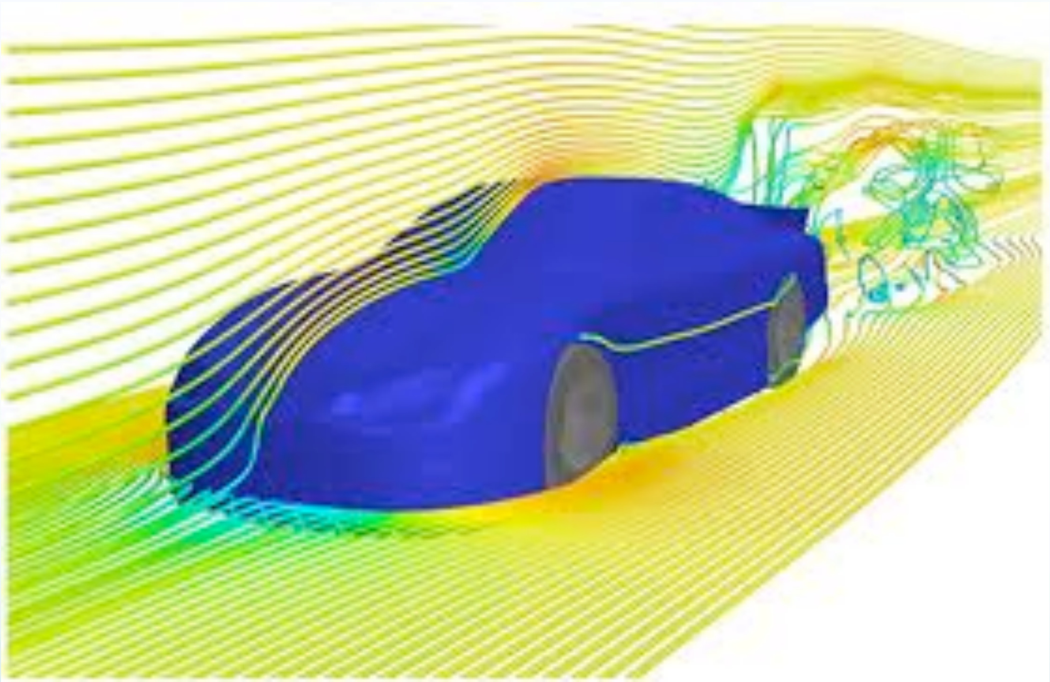
- A streamline is a line that is everywhere tangent to the velocity field – $dy/dx=v/u$ (governing equation)
- A streakline consists of all particles in a flow that have previously passed through a common point
- A pathline is the line traced out by a given particle as it flows
- For a steady flow they are all the same. For an unsteady flow they are not.

Example

- https://engineering.purdue.edu/~wassgren/applet/java/flowvis/
- http://www-mdp.eng.cam.ac.uk/web/library/enginfo/aerothermal_dvd_only/aero/fprops/cvanalysis/node8.html
- Look at these yourself – we will demonstrate an example using Matlab in a few slides.

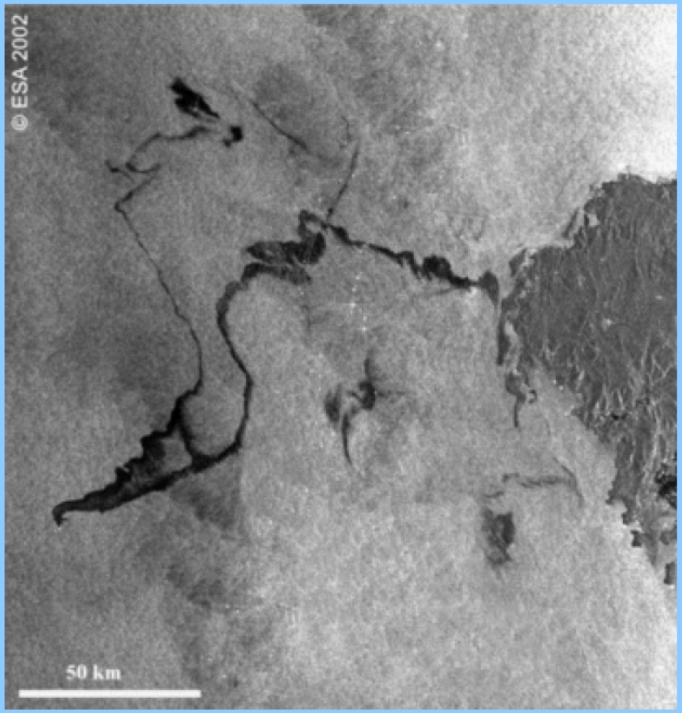
Streamlines

- Streamlines around a Nascar



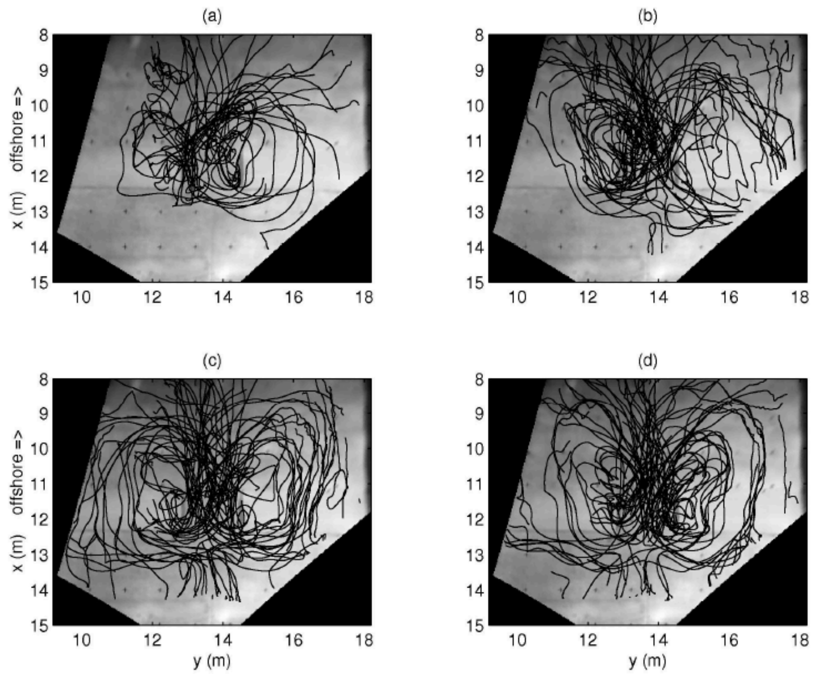
Streaklines

Streaklines From an Oil Spill off Spain



Pathlines

Pathlines from Rip Current



Example Problem

- Flow Above an Oscillating Plate with a vertical blowing is given by

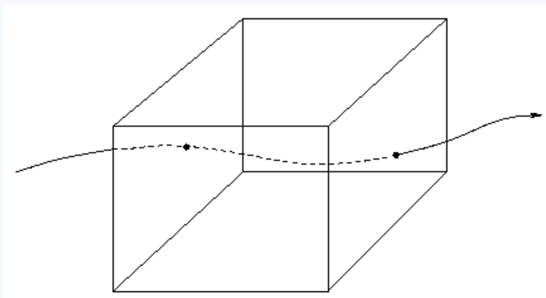
$$u = e^{-y}\cos(t - y) \qquad\qquad v = 1$$

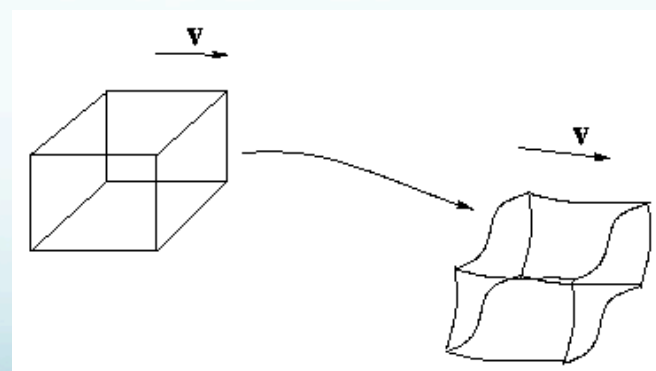
- Draw the streamlines at various times
- Draw pathlines
- Draw streaklines
- Compare to the steady case where $u = e^{-y}\cos(-y)$
- See Matlab code

Eulerian vs. Lagrangian Perpsective

- Eulerian
  - Sit and observe a fixed area from a fixed point
- Lagrangian
  - Travel with the flow and observe what happens around you
- Mixed – something that sits between the two

Eulerian vs. Lagrangian Perpsective

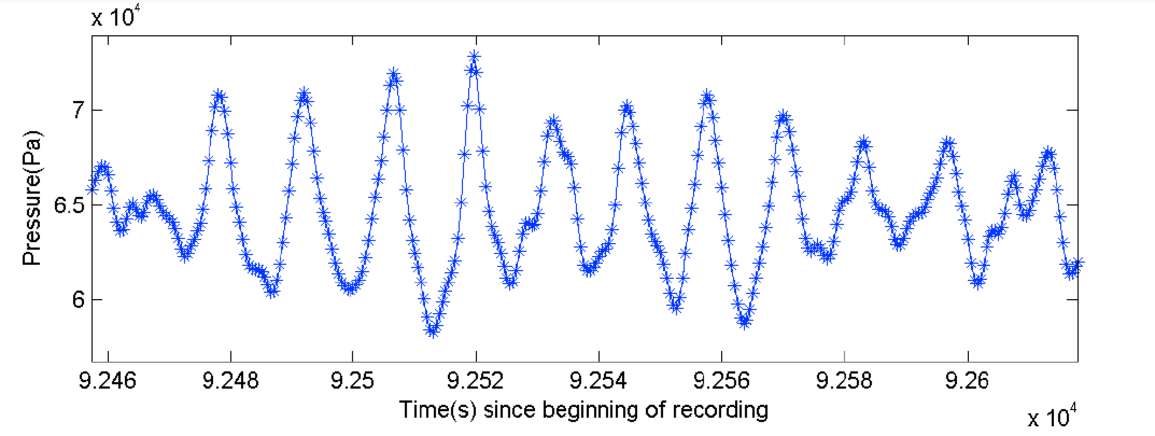




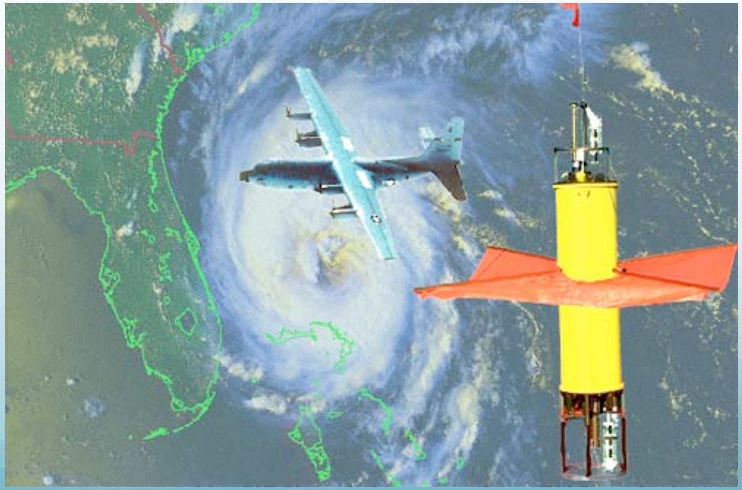
Eulerian vs. Lagrangian Perpsective – Which is which?

Experimental Measurements

- Fixed Measurement System



- A floating gauge



The Material Derivative

- Consider a fluid particle moving along its pathline (Lagrangian system)
- The velocity of the particle is given by

$$\mathbf{V}_A = \mathbf{V}_A(\mathbf{r}_A, t) = \mathbf{V}_A[x_A(t), y_A(t), z_A(t), t]$$

- It depends on the x,y, and z position of the particle
- Acceleration
  - $a_A = dV_A/dt$
    - It is tough to calculate this, but if we have an Eulerian picture......

The Material Derivative

$$\mathbf{a}_A = \frac{\partial \mathbf{V}_A}{\partial t} + u_A \frac{\partial \mathbf{V}_A}{\partial x} + v_A \frac{\partial \mathbf{V}_A}{\partial y} + w_A \frac{\partial \mathbf{V}_A}{\partial z}$$

- The material derivative (you can see it called the substantial derivative too) relates Lagrangian and Eulerian viewpoints and is defined as

$$\frac{D(\ )}{Dt} \equiv \frac{\partial(\ )}{\partial t} + u\frac{\partial(\ )}{\partial x} + v\frac{\partial(\ )}{\partial y} + w\frac{\partial(\ )}{\partial z}$$

- Or in compact notation

$$\frac{D(\ )}{Dt} = \frac{\partial(\ )}{\partial t} + (\mathbf{V} \cdot \nabla)(\ )$$

The Material Derivative

$$\frac{D(\ )}{Dt} \equiv \frac{\partial(\ )}{\partial t} + u\frac{\partial(\ )}{\partial x} + v\frac{\partial(\ )}{\partial y} + w\frac{\partial(\ )}{\partial z}$$

Unsteady local
Time derivative

Convective Effects

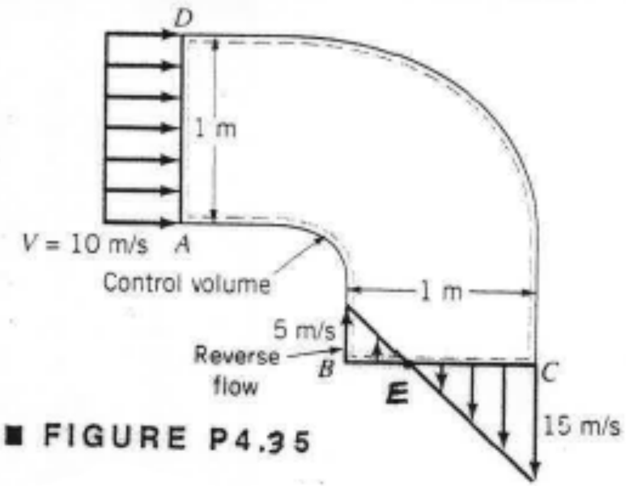
Example – convection of heat or a contaminant....

Control Volumes

- A system is a collection of matter of fixed identity (always the same packets)
- A Control Volume (CV) is a volume in space through which fluid can flow (it can be Lagrangian, i.e. moving and deforming with flow or Eulerian, i.e. fixed in space)
- CVs can be fixed, mobile, flexible, etc.
- All laws in continuum mechanics depart from a CV analysis (i.e. balance mass, momentum, energy etc in a sufficiently small control volume).

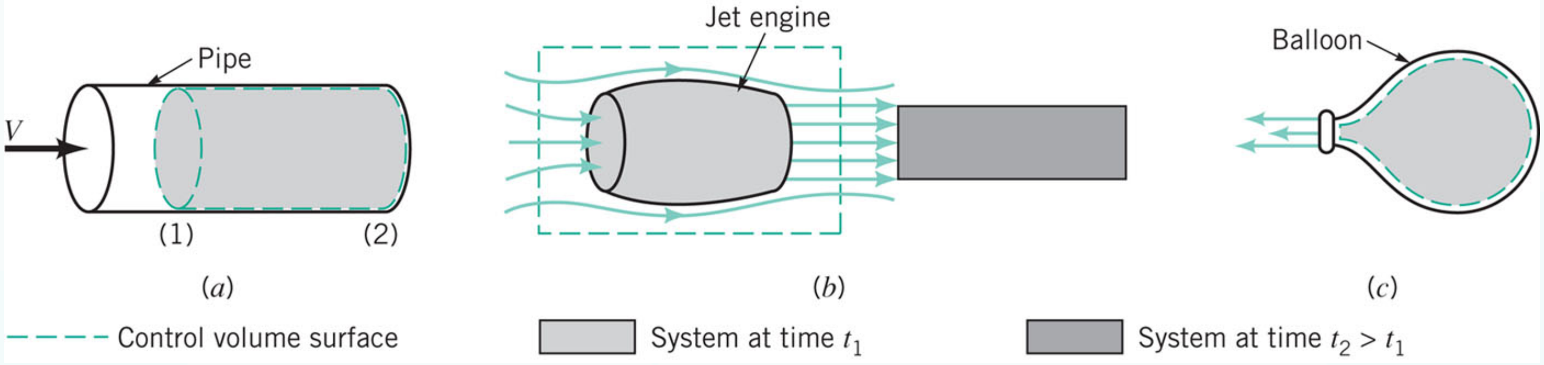
Sample Problem to distinguish System from Control Volume

4.35 Air enters an elbow with a uniform speed of 10 m/s as shown in Fig. P4.35. At the exit of the elbow the velocity profile is not uniform. In fact, there is a region of separation or reverse flow. The fixed control volume *ABCD* coincides with the system at time $t = 0$. Make a sketch to indicate **(a)** the system at time $t = 0.01$ s and **(b)** the fluid that has entered and exited the control volume in that time period.



■ FIGURE P4.35

Control Volumes



Reynolds Transport Theorem

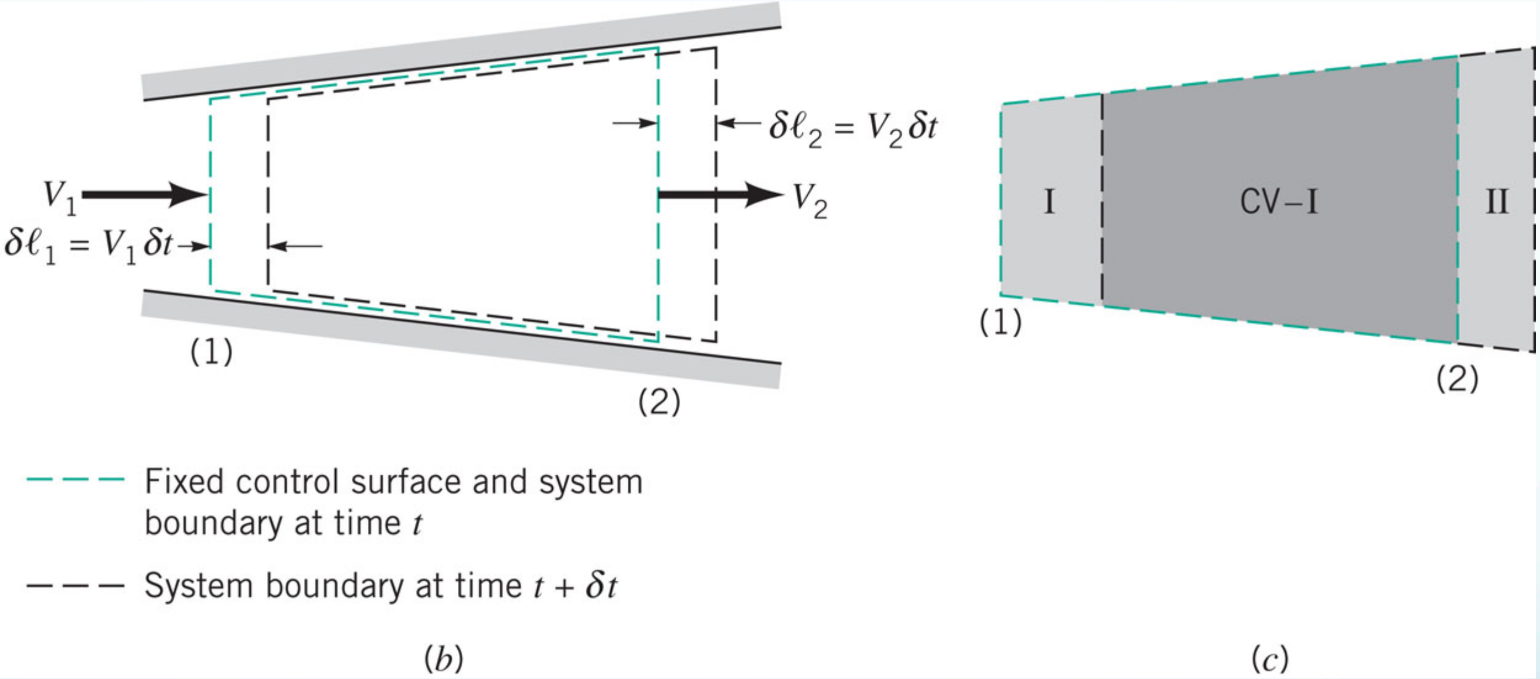
- A tool to relate system concepts to control volume concepts
- Let B be a fluid parameter (e.g. mass, temperature, momentum)
- Let b represent the amount of that parameter per unit mass

$$B = mb$$

- e.g. Momentum $B=mV$ => $b=V$

  Energy $B=1/2mV^2$ => $b=1/2\ V^2$

Reynolds Transport Theorem



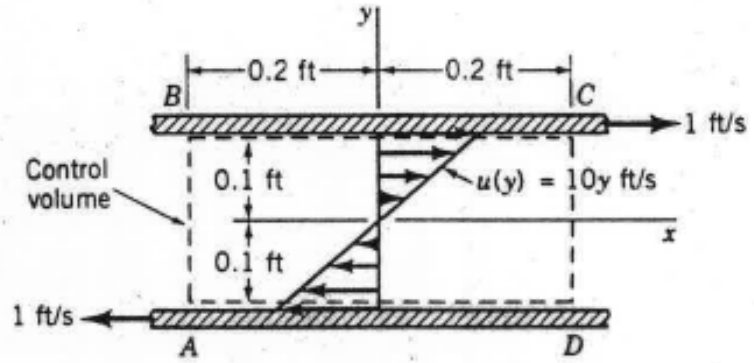
Reynolds Transport Theorem

- Generally written as

$$\frac{DB_{\text{sys}}}{Dt} = \frac{\partial B_{\text{cv}}}{\partial t} + \dot{B}_{\text{out}} - \dot{B}_{\text{in}}$$

Sample Problem

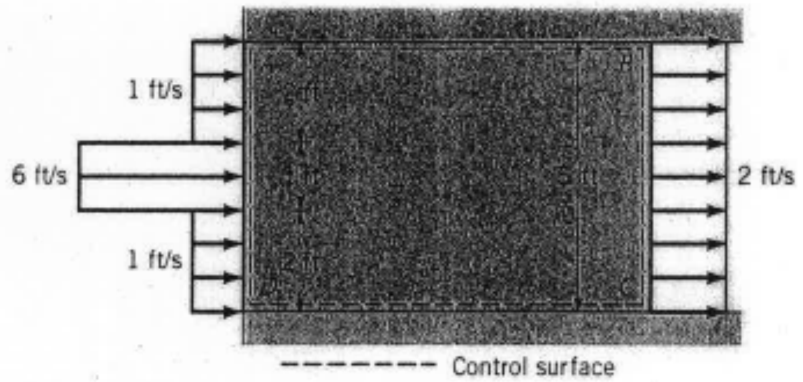
4.37 Two plates are pulled in opposite directions with speeds of 1.0 ft/s as shown in Fig. P4.37. The oil between the plates moves with a velocity given by $\mathbf{V} = 10y\hat{\mathbf{i}}$ ft/s, where y is in feet. The fixed control volume $ABCD$ coincides with the system at time $t = 0$. Make a sketch to indicate **(a)** the system at time $t = 0.2$ s and **(b)** the fluid that has entered and exited the control volume in that time period.



■ FIGURE P4.37

Sample Problem

4.39 Water enters a 5-ft-wide, 1-ft-deep channel as shown in Fig. P4.39. Across the inlet the water velocity is 6 ft/s in the center portion of the channel and 1 ft/s in the remainder of it. Farther downstream the water flows at a uniform 2-ft/s velocity across the entire channel. The fixed control volume *ABCD* coincides with the system at time $t = 0$. Make a sketch to indicate **(a)** the system at time $t = 0.5$ s and **(b)** the fluid that has entered and exited the control volume in that time period.



■ FIGURE P4.39