CE30460 - Fluid Mechanics

Diogo Bolster

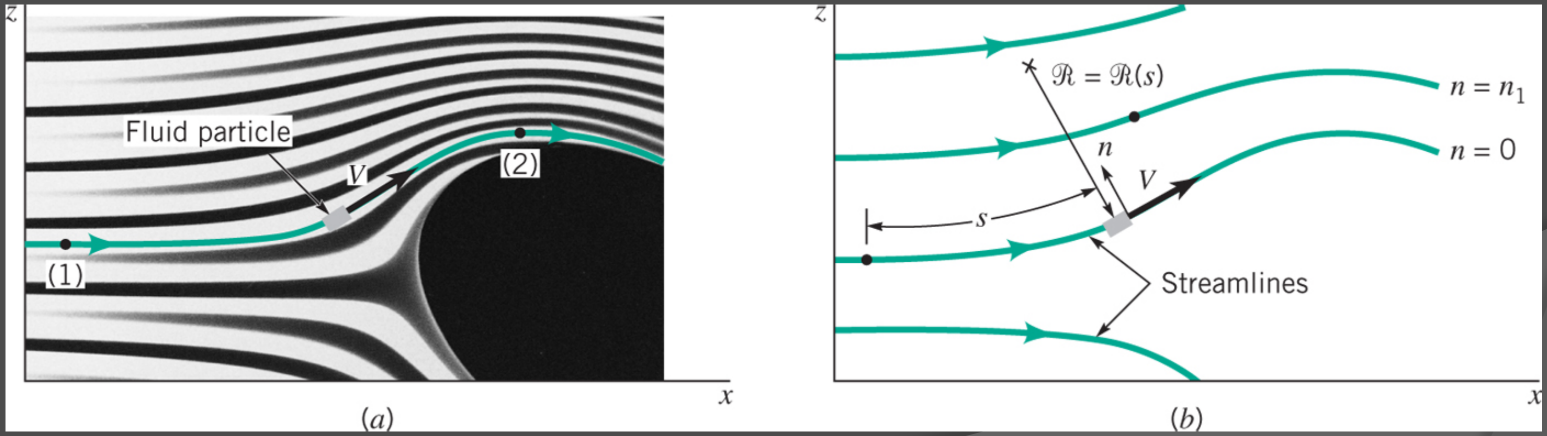
CHAPTER 3 – BERNOULLI'S EQUATION

Newton's Second Law

- F=ma
- What does this mean for a fluid? (inviscid)
- First we need to understand streamline:
  - If a flow is steady a streamline depicts the path a fluid particle will take through the flow. They are tangent to the velocity vectors
    - http://www.youtube.com/watch?v=j6yB90vno1E&feature=related
    - http://www.youtube.com/watch?v=rbMx2NMqyuI&feature=related
    - http://www.youtube.com/watch?v=0xsC-UVFjps&NR=1
  - The are between two streamlines is called a streamtube

Streamlines

- We can calculate acceleration along an normal to a streamline
- Note – particles travel along a streamlines, but that does not mean a constant velocity
- Two components
  - Along the streamline
  - Normal to streamline



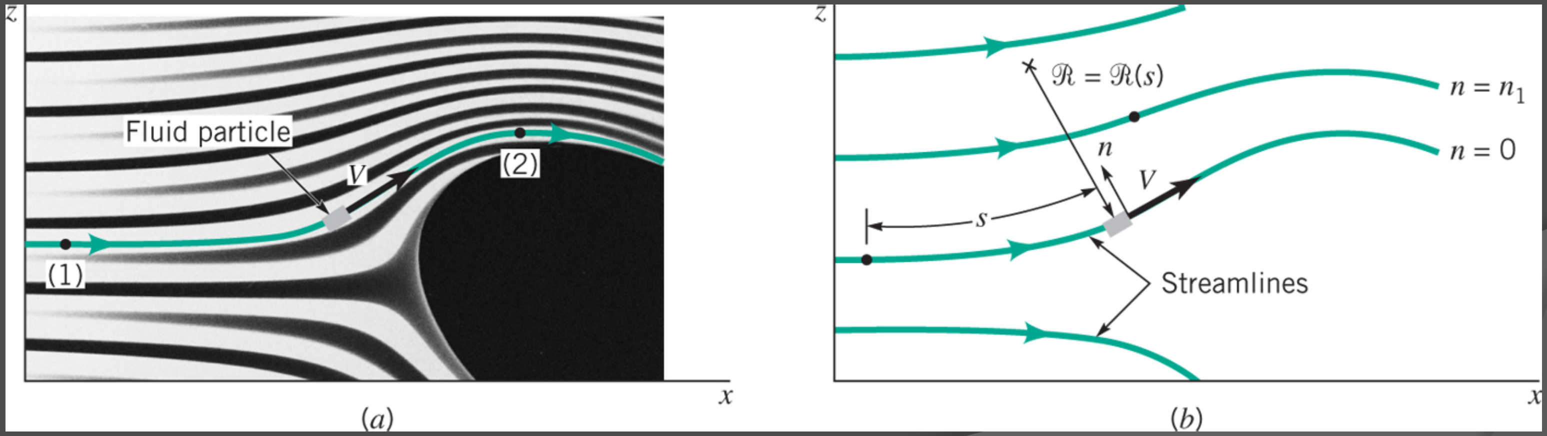
Streamlines

- We can calculate acceleration along an normal to a streamline
- Note – particles travel along a streamlines, but that does not mean a constant velocity
- Two components
  - Along the streamline
  - Normal to streamline (centrifugal)

$a_s = dV/dt = (\delta V/\delta s)(ds/dt) = (\delta V/\delta s)V$

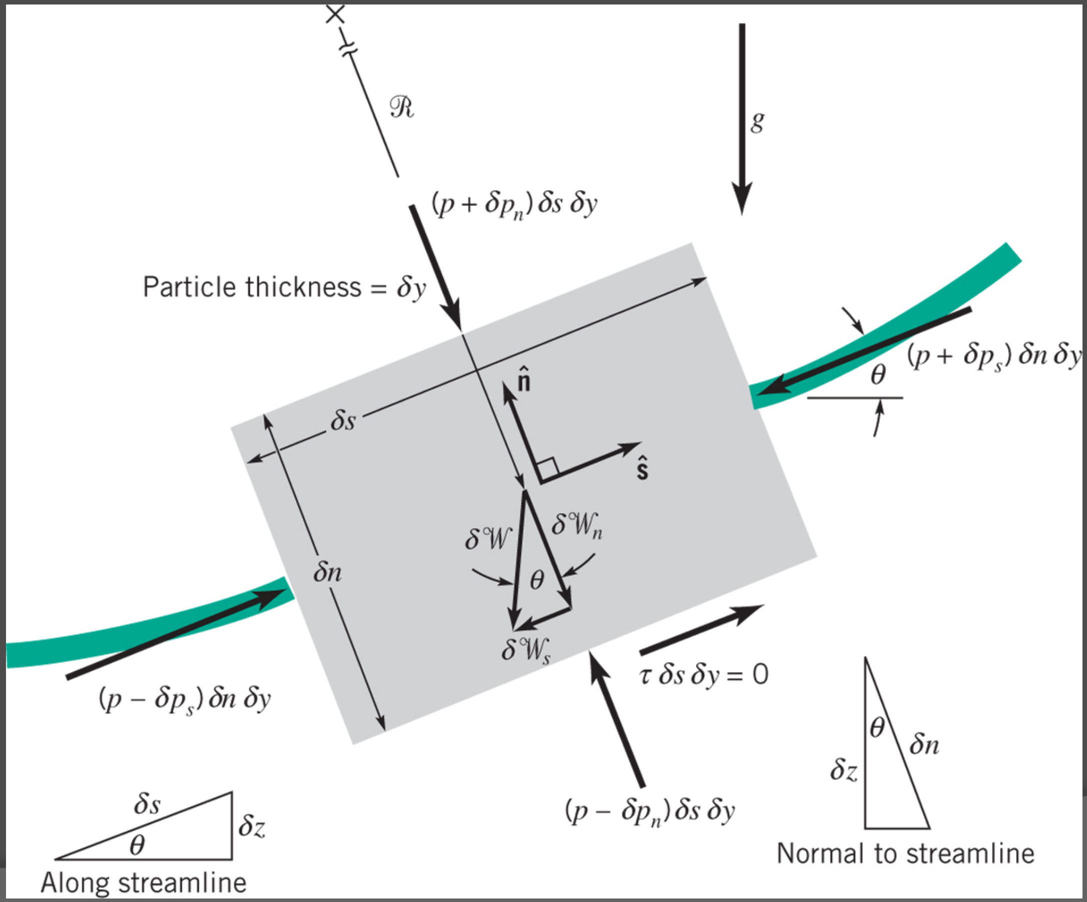
$a_n = V^2/\mathscr{R}$

$\mathscr{R}$ is the local radius of curvature



Newton's Second Law along a streamline

- F=ma – apply it to the small control volume depicted here (along a streamline!!)



$$-\gamma \sin\theta - \frac{\partial p}{\partial s} = \rho V \frac{\partial V}{\partial s}$$

The Bernoulli Equation

- Newton's second law for an inviscid fluid is called the Bernoulli equation. It is technically a momentum equation, but often refers to energy…

$$dp + \frac{1}{2}\rho d(V^2) + \gamma\, dz = 0 \qquad \text{(along a streamline)}$$

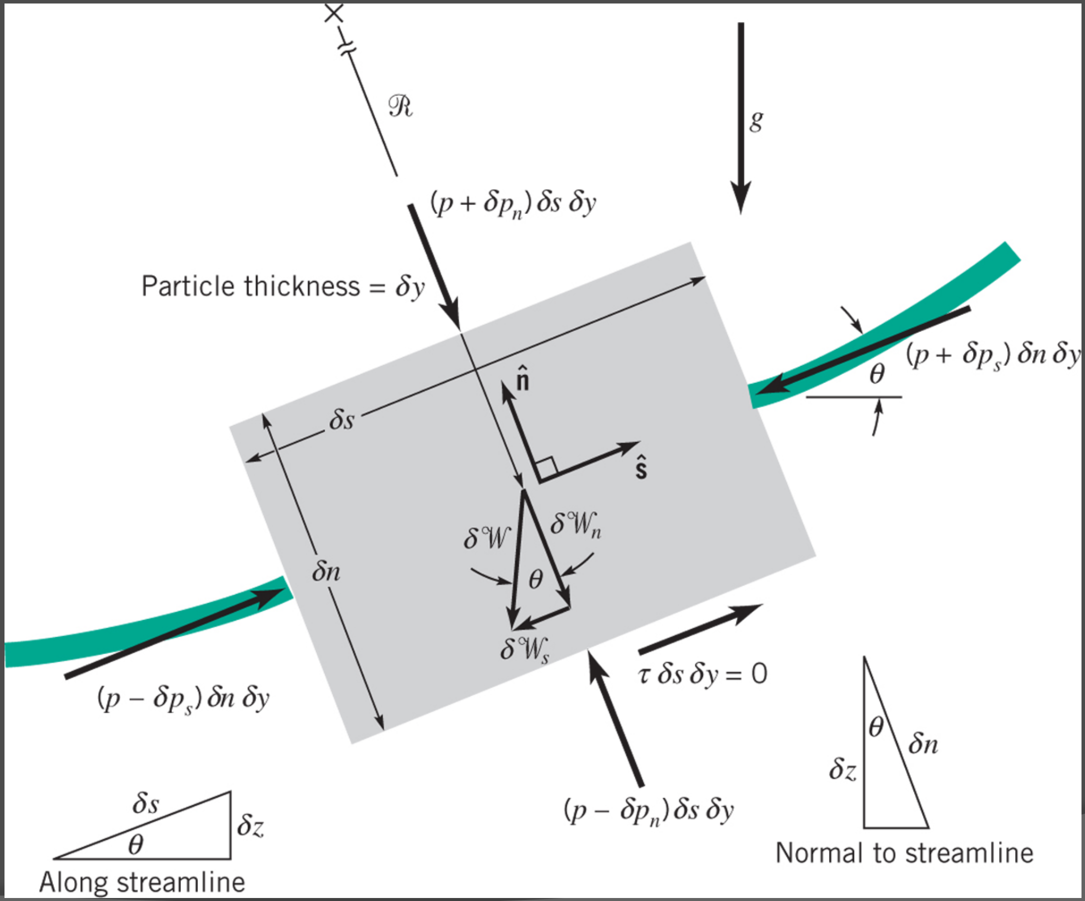
Pressure Acceleration Gravity

- Integrate assuming constant density and we obtain

$$p + \tfrac{1}{2}\rho V^2 + \gamma z = \text{constant along streamline}$$

Normal to a Streamline?

- Go back and do the balance



$$-\gamma \frac{dz}{dn} - \frac{\partial p}{\partial n} = \frac{\rho V^2}{\mathcal{R}}$$

Normal to a Streamline?

$$-\gamma \frac{dz}{dn} - \frac{\partial p}{\partial n} = \frac{\rho V^2}{\mathcal{R}}$$

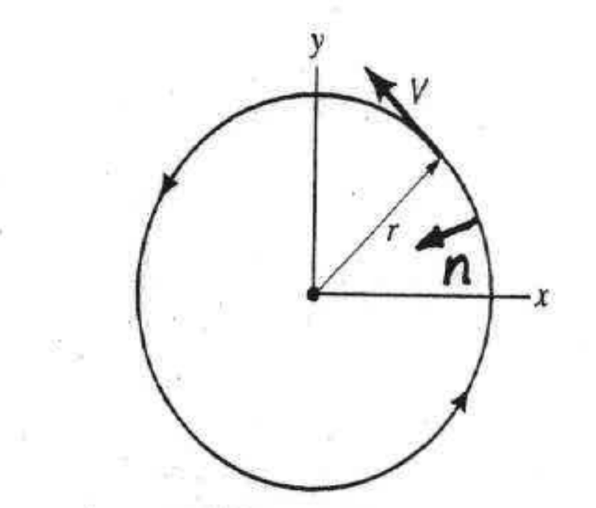
$$p + \rho \int \frac{V^2}{\mathcal{R}}\, dn + \gamma z = \text{constant across the streamline}$$

Sample Problem 1

- What pressure gradient along a streamline, dp/ds, is required to accelerate water upward in a vertical pipe at a rate of 10 m/s^2? What is the answer if the flow is downwards?

Sample Problem 2

- Consider a tornado. The streamlines are circles of radius r and speed V. Determine pressure gradient dp/dr for
- (a) water r=10cm in and V=0.2 m/s
- (b) air r=100 m and V=200 km/h



Back to Bernoulli Equation

- A very important equation

$$p + \tfrac{1}{2}\rho V^2 + \gamma z = \text{constant along streamline}$$

- The above version is called the energy representation
- But what does it actually mean? Let's rewrite it a little

$$\frac{p}{\gamma} + \frac{V^2}{2g} + z = \text{constant on a streamline}$$

Physical Interpretation

$$\frac{p}{\gamma} + \frac{V^2}{2g} + z = \text{constant on a streamline}$$

Pressure Head

Velocity Head

Elevation Head

Each of these terms represents a potential that the fluid has and that can freely be exchanged along a streamline, e.g. if velocity decreases either the pressure head of elevation head must increase

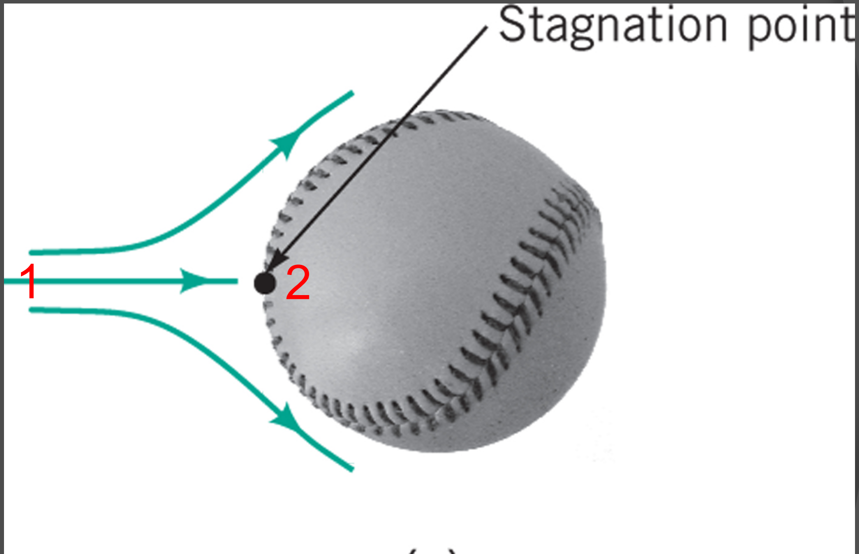
Static, Dynamic and Hydrostatic Pressure

$$p + \frac{1}{2}\rho V^2 + \gamma z = \text{constant along streamline}$$

- Static Pressure
  - The pressure one feels if static relative to the flow (i.e. moving with the flow). This is the pressure in the Bernoulli equation
    - p
- Dynamic Pressure
  - The pressure exerted by the velocity field – i.e. associated with the second term in Bernoulli equation
    - $\rho V^2/2$
- Hydrostatic Pressure
  - The pressure from the third term in Bernoulli
    - γz

Stagnation Pressure

- Stagnation Point
- Consider the situation where z=const (e.g. baseball to right).
- Consider a frame of reference where fluid moves and baseball stationary (relative velocity)
- Point 1 is far from the ball
- Point 2 is on the ball surface and the same streamline as 1.



(a)

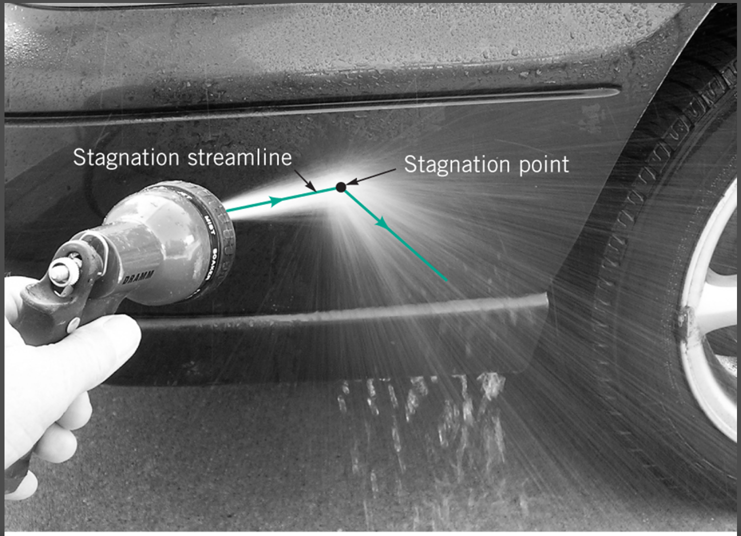
$$p_2 = p_1 + \frac{1}{2}\rho V_1^{\,2}$$

- The pressure at a stagnation point is called : Stagnation Pressure.

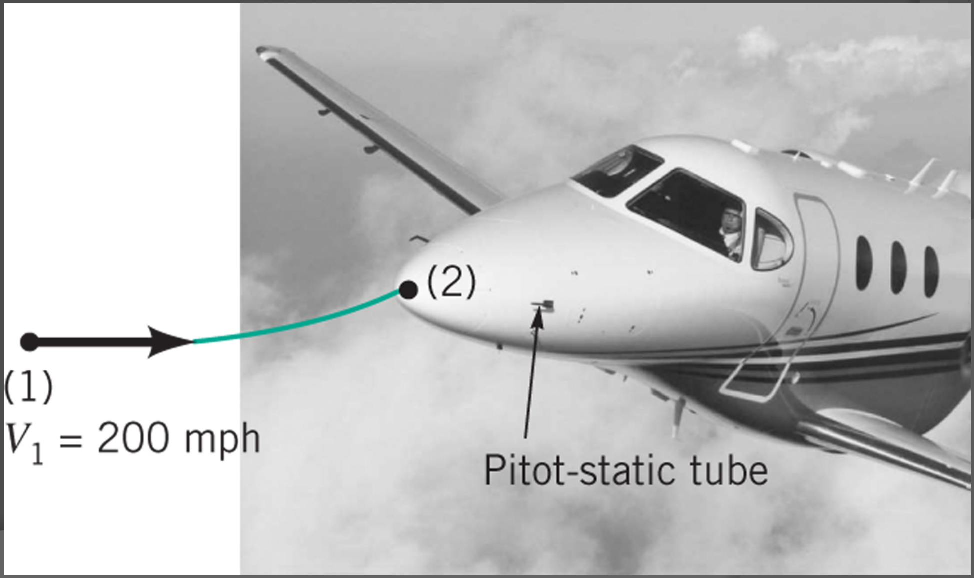
Total Pressure

- In analogy to the stagnation pressure we can define a total pressure (i.e. if you convert all you dynamic and hydrostatic pressure to a static one).

$$p_T = p_1 + \frac{1}{2}\rho V_1^2 + \gamma z$$



(b)



Sample Problem 1

3.20 Some animals have learned to take advantage of the Bernoulli effect without having read a fluid mechanics book. For example, a typical prairie dog burrow contains two entrances—a flat front door, and a mounded back door as shown in Fig. P3.20. When the wind blows with velocity V_0 across the front door, the average velocity across the back door is greater than V_0 because of the mound. Assume the air velocity across the back door is $1.07V_0$. For a wind velocity of 6 m/s, what pressure differences, $p_1 - p_2$, is generated to provide a fresh air flow within the burrow?

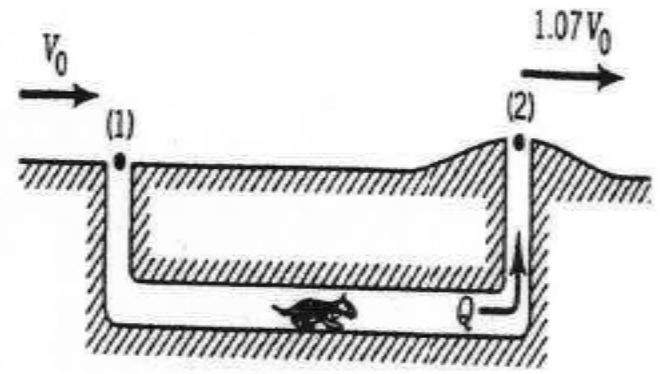
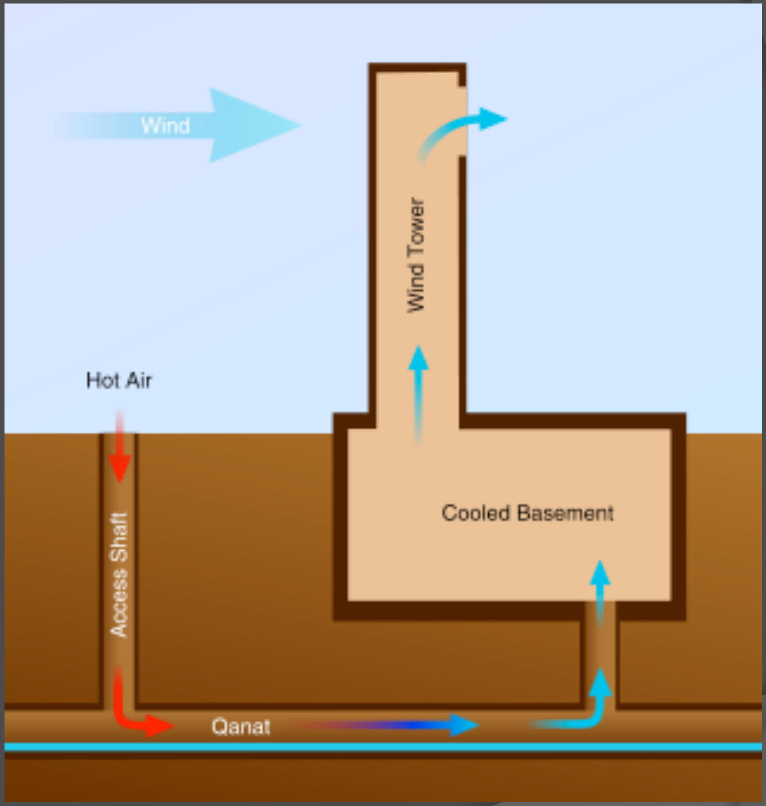


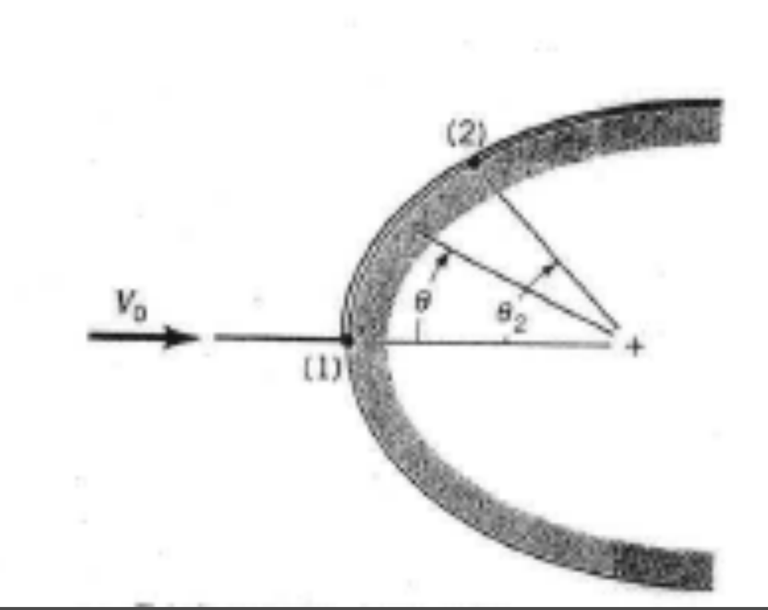
FIGURE P3.20

Application - Iranian Wind Towers



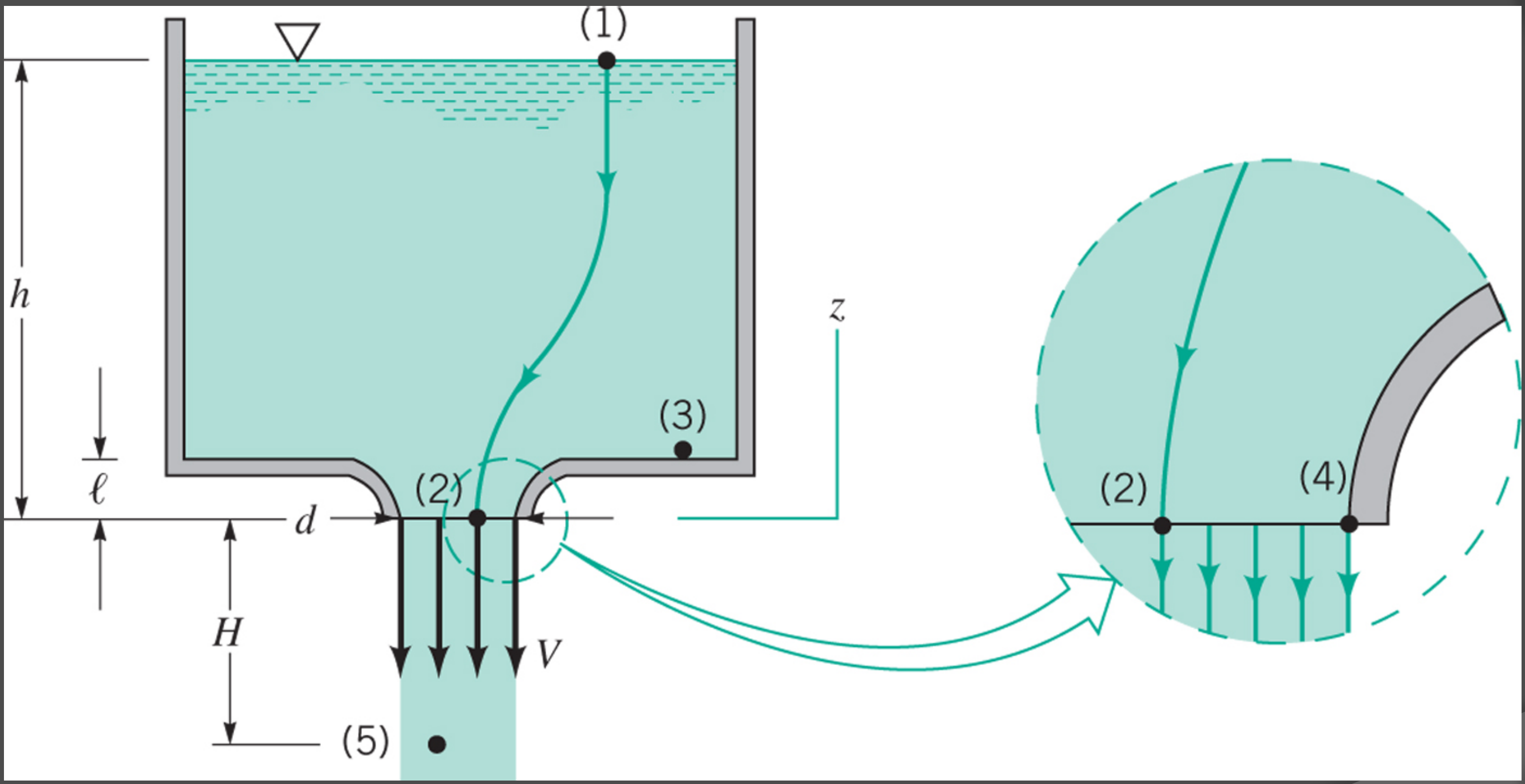
http://nd.edu/~bolster/Research%20Highlights.html

Sample Problem 2

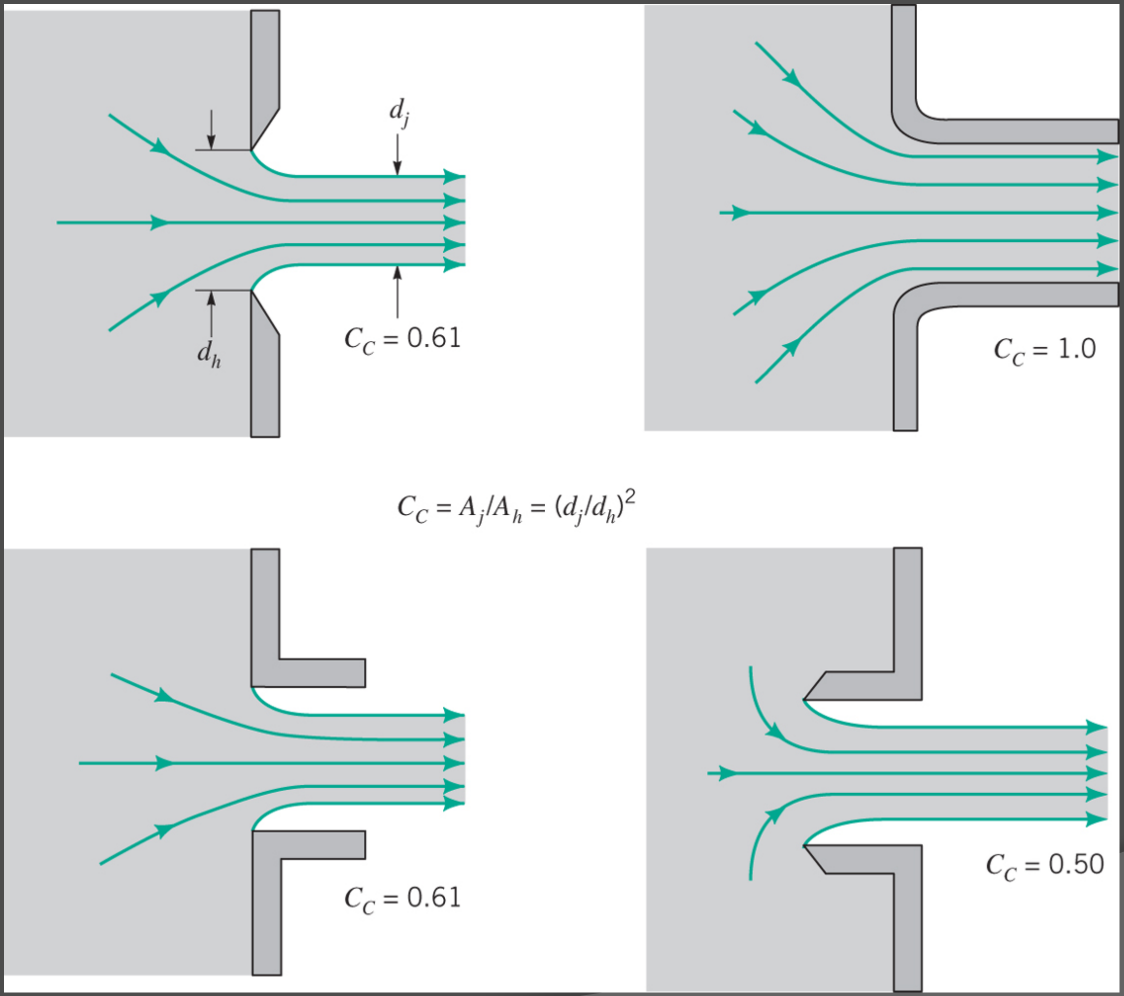


- An inviscid fluid flows along the stagnation streamline with starting velocity V_0.
- Upon leaving the stagnation point the velocity along the surface of the object is assumed to be 2V0 sin ().
- At what angular position should a hole be drilled to give a pressure difference of p_1-p_2=$\rho V_0^2/2$?

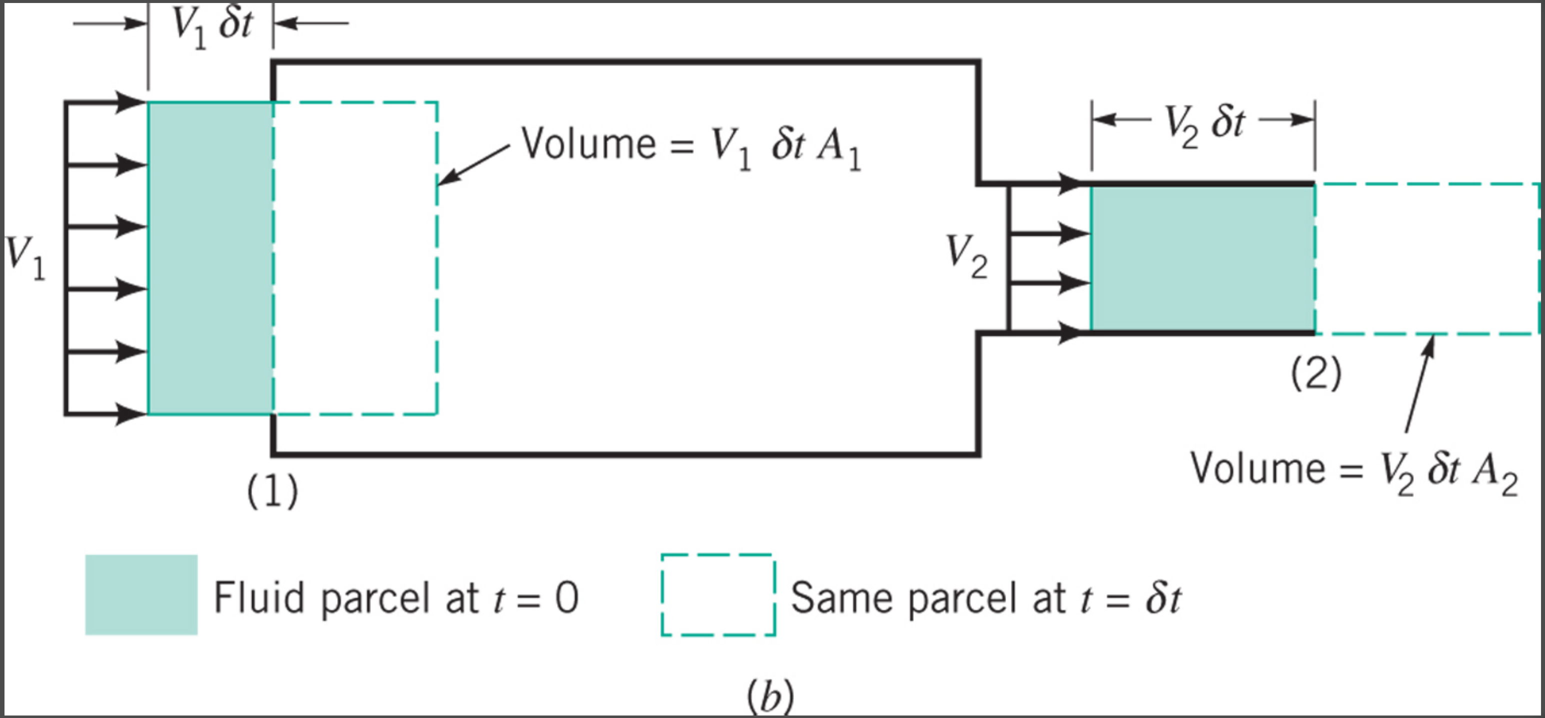
Using the Bernoulli Equation 1 – Free Jet



Correction factor – Contraction Coefficient

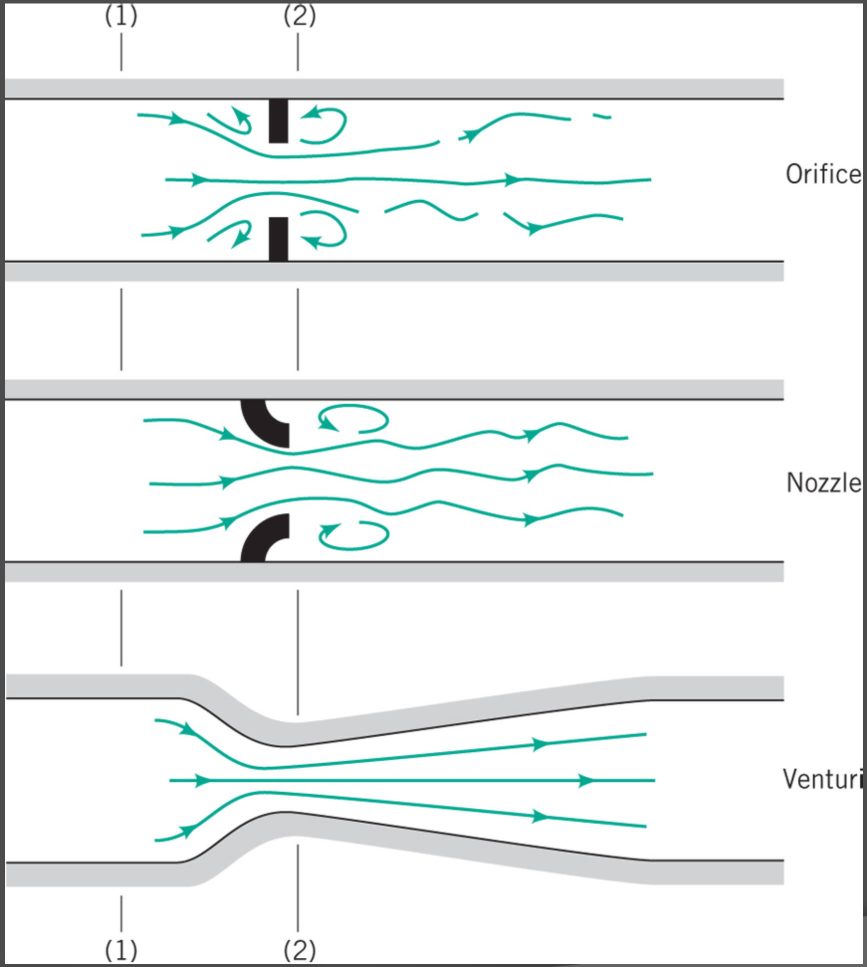


Using the Bernoulli Equation 2 – Confined Flows



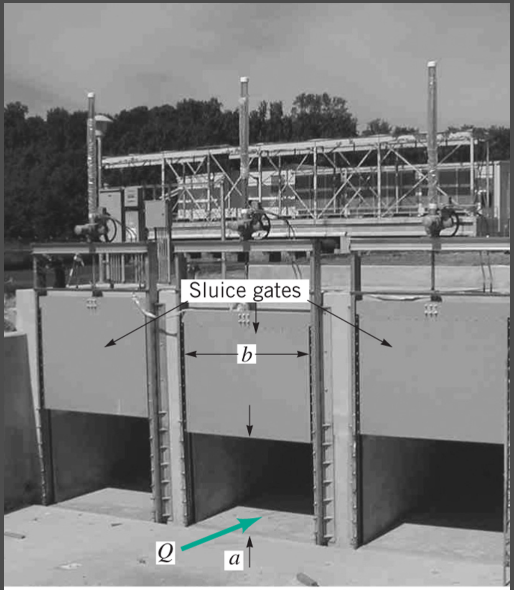
$$A_1V_1 = A_2V_2, \text{ or } Q_1 = Q_2$$

Using the Bernoulli Equation 3 – Flowrate Measurement

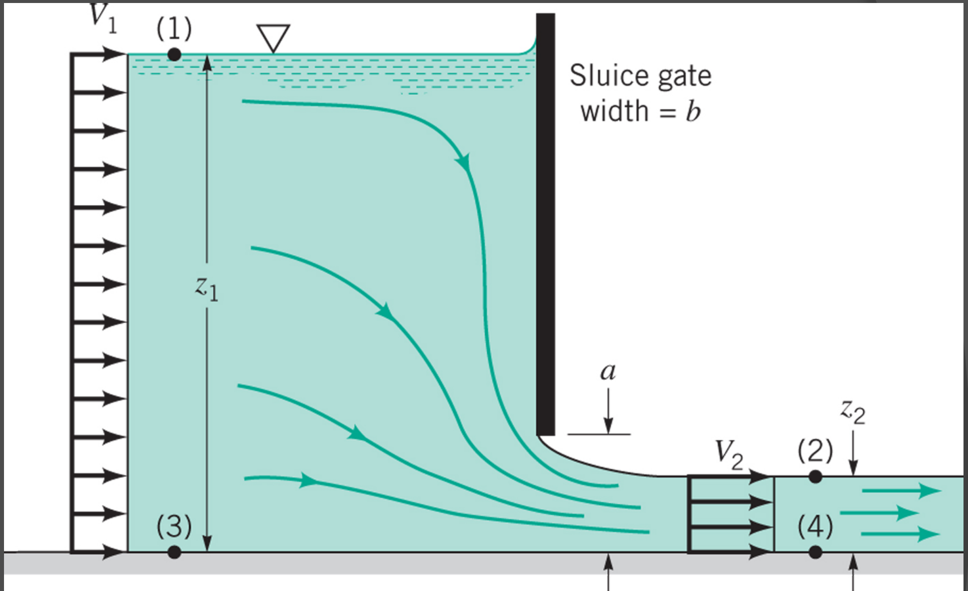


$$Q = A_2\sqrt{\frac{2(p_1 - p_2)}{\rho[1 - (A_2/A_1)^2]}}$$

Sluice Gate (Open Channel Flowmeter)



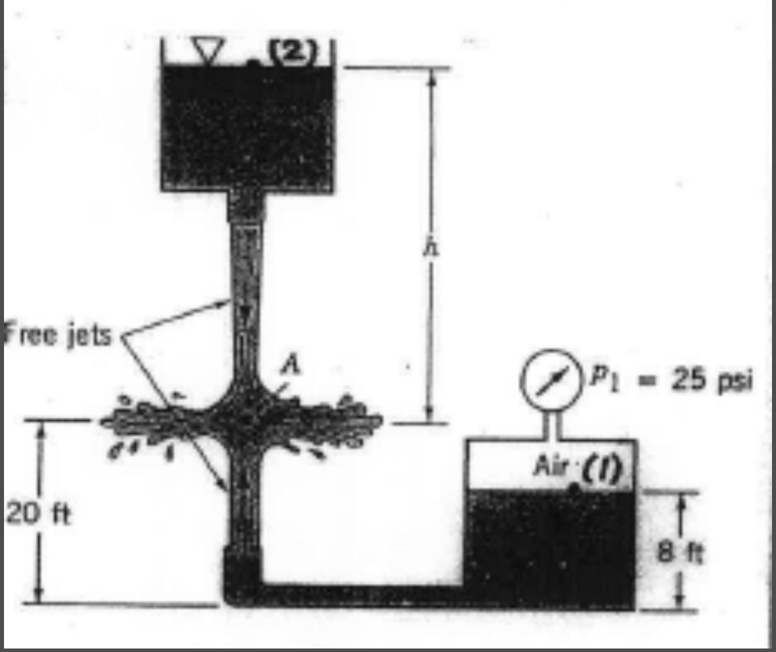
(a)



(b)

$$Q = z_2 b \sqrt{\frac{2g(z_1 - z_2)}{1 - (z_2/z_1)^2}}$$

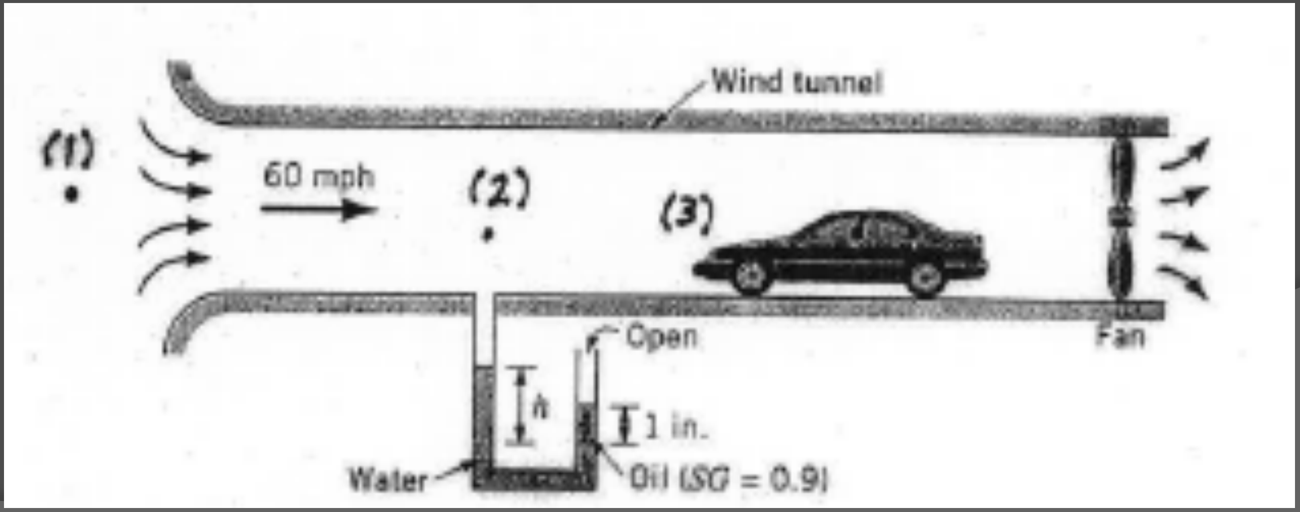
Sample Problem 1



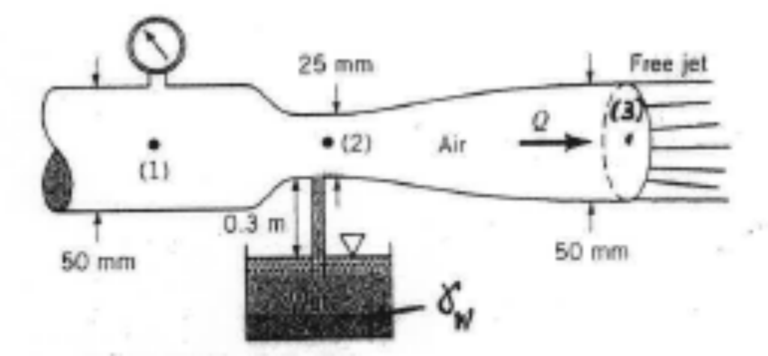
- Streams of water from two tanks impinge on one another as shown
- Neglect viscosity
- Point A is a stagnation point
- Determine h

Sample Problem 2

- Air is drawn in a wind tunnel for testing cars
- Determine the manometer reading h when velocity in test section is 60 mph
- Note that there is a 1 inch column of oil on the water
- Determine the difference between the stagnation pressure on the front of the automobile and the pressure in the test section



Sample Problem 3



- Air flows through this device
- If the flow rate is large enough the pressure at the throat is low enough to draw up water. Determine flow rate and pressure needed at Point 1 to draw up water at point 2
- Neglect compressibility and viscosity

Restrictions of the Bernoulli Equation

- Incompressible
- Invisicid (can be corrected for with viscous losses term – see later on in course)
- Does not account for mechanical devices (e.g. pumps) – again can be corrected for as above.
- Apply along a streamline!

Useful Equations

- Streamwise Acceleration $a_s = V\frac{\partial V}{\partial s}, \qquad a_n = \frac{V^2}{\mathcal{R}}$
- Bernoulli Equation $p + \frac{1}{2}\rho V^2 + \gamma z = \text{constant along streamline}$
- Pitot Static Tube $V = \sqrt{2(p_3 - p_4)/\rho}$
- Free Jet $V = \sqrt{2\frac{\gamma h}{\rho}} = \sqrt{2gh}$
- Flow Meter $Q = A_2\sqrt{\frac{2(p_1 - p_2)}{\rho[1 - (A_2/A_1)^2]}}$
- Sluice Gate $Q = z_2 b\sqrt{\frac{2g(z_1 - z_2)}{1 - (z_2/z_1)^2}}$