Homework 2

(1) $\frac{\delta C}{\delta t} = \frac{\delta^2 C}{\delta x^2}$ $\quad 0 < x < 20 \quad$ $C(x, t=0) = \delta(x-10)$

$$\left.\frac{\delta C}{\delta x}\right|_{x=0} = \left.\frac{\delta C}{\delta x}\right|_{x=20} = 0$$

Polyanin tells us the solution to this is

$$\int_0^{20} \delta(\xi - 10)\, G(x, \xi, t)\, d\xi \qquad G(x,\xi,t) = \frac{1}{20} + \frac{2}{20} \sum_{n=1}^{\infty} \cos\left(\frac{n\pi x}{\ell}\right) \cos\left(\frac{n\pi \xi}{\ell}\right) e^{-\frac{n^2\pi^2 t}{20}}$$

(i) $[L]$ = Length
$[D]$ = Length^2 Time^{-1} } Combine to get $\tau = L^2/D$
→ only natural timescale for parameters

(ii) See figures + code

$$C(x,t) = \frac{1}{20} + \frac{1}{10} \sum_{n=1}^{\infty} \cos\left(\frac{n\pi x}{20}\right) \cos\left(\frac{n\pi\, 10}{20}\right) e^{-\frac{n^2\pi^2 t}{20}}$$

When $C(t=0) = \delta(x-15)$

$$C(x,t) = \frac{1}{20} + \frac{1}{10} \sum_{n=1}^{\infty} \cos\left(\frac{n\pi x}{20}\right) \cos\left(\frac{n\pi\, 15}{20}\right) e^{-\frac{n^2\pi^2 t}{20}}$$

(2) $\frac{\delta c}{\delta t} = D \frac{\delta^2 c}{\delta x^2}$ $\quad 0 < x < \infty$

$C(x, t=0) = 0 \quad C(x=0, t) = C_0$

From Polyanin

(i) $C(x,t) = \int_0^t C_0 \, H(x, t-\tau) \, d\tau \quad H(x,t) = \frac{x}{2\sqrt{\pi D}\, t^{3/2}} e^{-x^2/4Dt}$

Evaluate this integral with Mathematica for example

$$C(x,t) = C_0\left(1 - \text{erf}\left(\frac{x}{\sqrt{4Dt}}\right)\right)$$

or

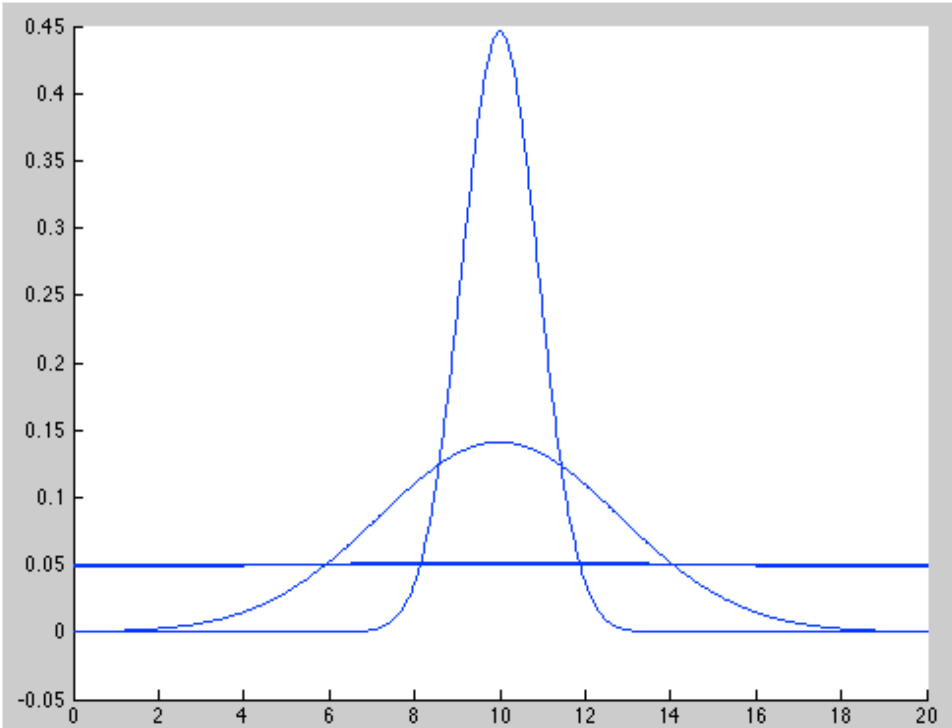
$$= C_0 \, \text{erfc}\left(\frac{x}{\sqrt{4Dt}}\right)$$

(ii) $M(t) = \int_0^\infty C(x,t)\, dx = \frac{2}{\sqrt{\pi}} \sqrt{Dt}$

(iii) $M \propto \sqrt{D} \Rightarrow$ double D means $\sqrt{2}$ more mass

%Problem1.m

```
clear
clc
close all

x=linspace(0,20,1000);

D=1;
L=20;

tau=L^2/D;

%for C(t=0)=delta(x-10)

for t=[0.001 0.01 0.1 1 10]*tau;
   for ll=1:length(x)

       n=1:100;
       sm=cos(n*pi*x(ll)/L).*cos(n*pi*10/L).*exp(-D*n.^2*pi^2*t/L^2);

       w(ll)=1/L+2/L*sum(sm);


   end
   figure(1)
   hold on
   plot(x,w)
end

%for C(t=0)=delta(x-15)

for t=[0.001 0.01 0.1 1 10]*tau;
   for ll=1:length(x)

       n=1:101;
       sm=cos(n*pi*x(ll)/L).*cos(n*pi*15/L).*exp(-D*n.^2*pi^2*t/L^2);

       w(ll)=1/L+2/L*sum(sm);


   end
   figure(2)
   hold on
   plot(x,w)
end
```


0.45
0.4
0.35
0.3
0.25
0.2
0.15
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0.05
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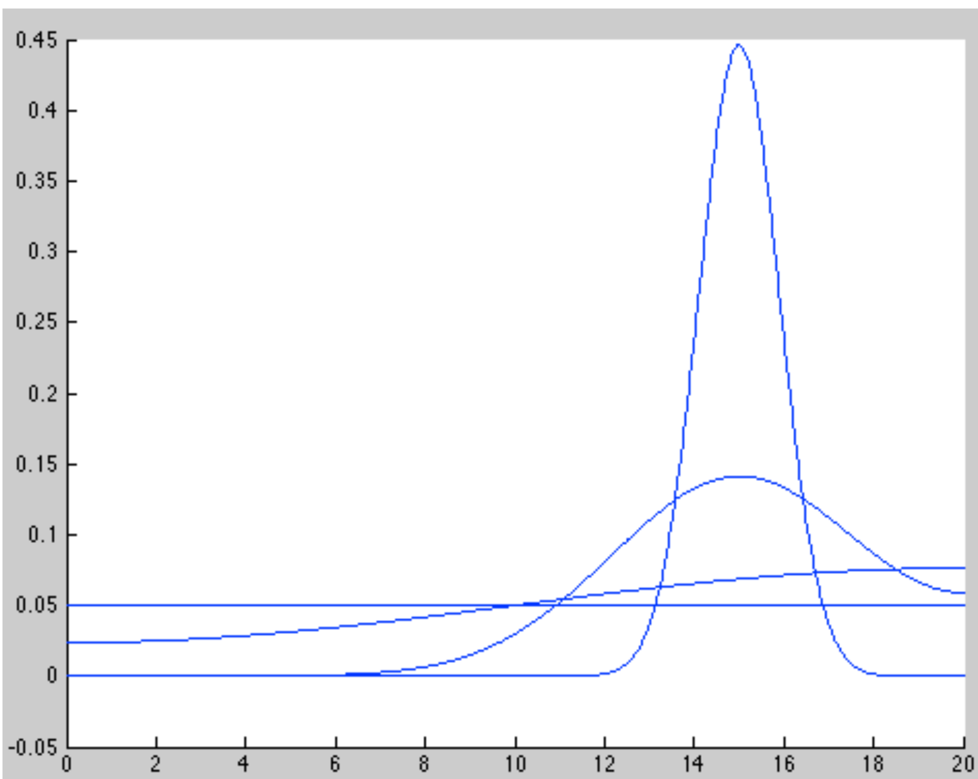
20


0.45
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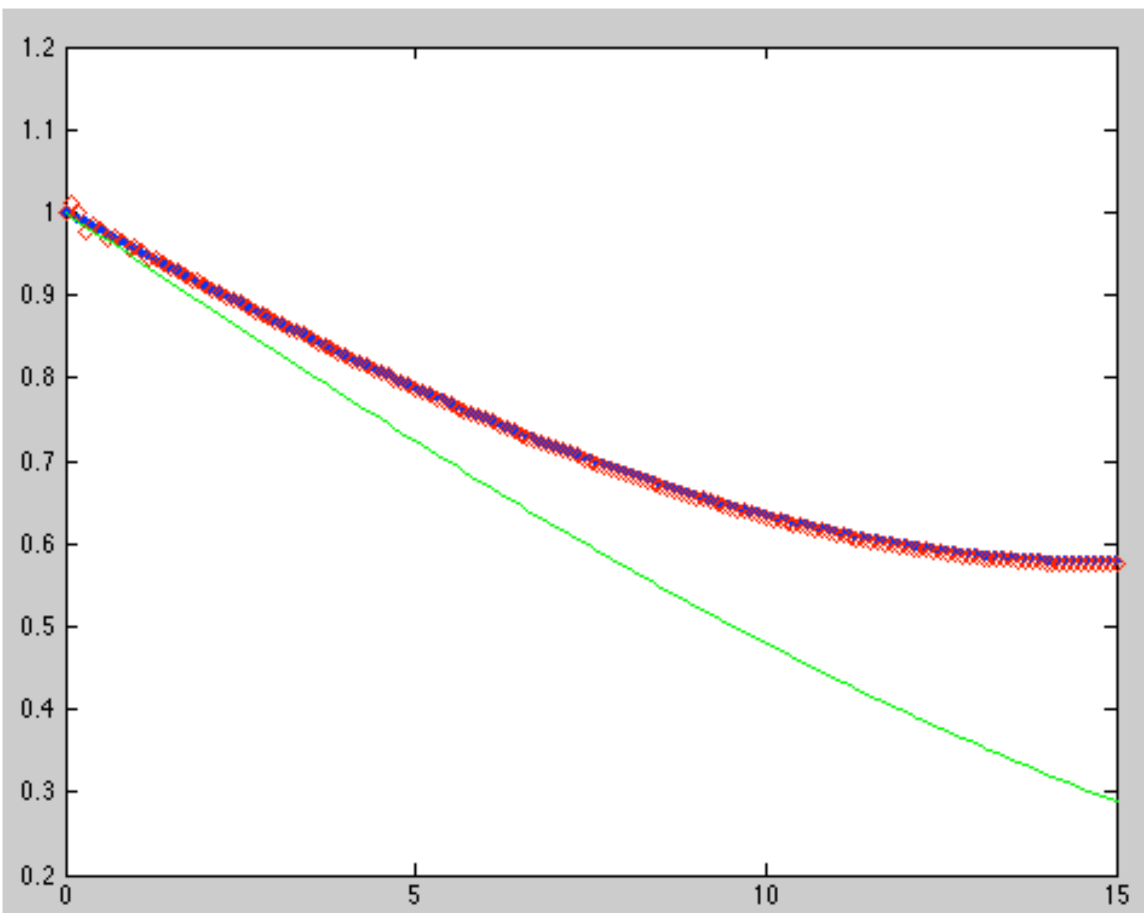
%Problem_3.m

```
clear
clc
close all

x=0:0.1:15;  %Define your spatial domain
dx=x(2)-x(1);  %spatial deicretization;
dt=0.001;       %time step
Nsteps=100000;    %Number of Timesteps
D=1;       %Diffusion Coefficient

C=zeros(size(x));  %Our Initial Concentration
C(1)=1;           %The Boundary Condition


for j=1:Nsteps

   %update concentration based no finite differences. We only update
   %locations 2 through N-1 as the end are determined from the boundary
   %conditions.

   Cnew(2:length(x)-1)=C(2:length(x)-1)+D*dt/dx^2*(C(1:length(x)-2)-
2*C(2:length(x)-1)+C(3:length(x)));

   %Boundary Conditions
   Cnew(1)=1;
   Cnew(length(x))=Cnew(length(x)-1);

   %Update concentration field
   C=Cnew;

end

plot(x,C,'.')
hold on
%Greens function - finite domain.

n=0:1000; %number of terms in expansion

t=Nsteps*dt;
l=max(x);

for kk=1:length(x)
```

```
    sm=4*(1-exp(-
D.*pi^2.*(1+2*n).^2/4/l^2*t))./pi./(1+2*n).*sin((1+2*n)*pi*x(kk)/2/l);

   Cg(kk)=sum(sm);

end
Cg(1)=1;

hold on
plot(x,Cg,'rd')

%seminfite domain

Csm=erfc(x/sqrt(4*D*t));
plot(x,Csm,'g')
```



```
%Problem 4
tic

clear
clc
%close all

%define your boundaries
xlow=0;
xup=20;

dt=.01; %timestep
D=1;    %diffusion coefficient
N=1e6; % number of particles

xinit=10; %initial location of all the particles

x=xinit*ones(1,N);   %initial condition4
Nsteps=1e3;

for kk=1:Nsteps

    kk

    x=x+sqrt(2*D*dt)*randn(size(x));

    %reflect
    apple=find(x<xlow);
    x(apple)=-x(apple);

    apple=find(x>xup);
    x(apple)=2*xup-x(apple);
end


[c,x]=hist(x,100);
dx=x(2)-x(1);
plot(x,c/sum(c)/dx,'k')
toc
```

The following subplot shows concentrations for various numbers of particles with the above code. Notice as how particle numbers increase solution becomes smoother and looks much more like that in Problem 1.