HW 2

① $s\tilde{c}_1 - \cancel{c_0}^{0} + u\frac{\partial \tilde{c}_1}{\partial x} = -\alpha\tilde{c}_1 + \alpha\tilde{c}_2$

$$\beta s\tilde{c}_2 - \beta \underbrace{c_{20}}_{\delta(x)} = \alpha\tilde{c}_1 - \alpha\tilde{c}_2$$

$$\tilde{c}_2 = \frac{\alpha\tilde{c}_1 + \beta\delta(x)}{\beta s + \alpha}$$

$$\therefore \quad s\tilde{c}_1 + u\frac{\partial \tilde{c}_1}{\partial x} = -\alpha\tilde{c}_1 + \frac{\alpha^2\tilde{c}_1 + \alpha\beta\delta(x)}{s+\alpha}$$

Solve in Mathematica $\Rightarrow$

$$\tilde{c}_1 = \frac{\alpha\beta}{(\alpha+\beta s)u} e^{-\frac{\alpha s - \alpha\beta s - \beta s^2}{(\alpha+\beta s)u}x}$$

Invert using Matlab routine

When $\beta = 1$ similar to $c_1(t=0) = \delta(x)$ from class
because α is large

$\beta = 10 \Rightarrow$ concentrations are 10 times bigger because
more mass in initial condition.

(2) If you look at data peak in BTC arrives about 1.5 later than by advection in mobile region alone

$$\tau = \frac{x}{u} = \frac{50}{2} = 25\ s$$

$$t_{peak} \approx 37.5\ s$$

$\therefore$ A good guess for $\beta = 0.5$

Playing with α you find $\alpha = 0.4\ s^{-1}$ matches data

(3) From class notes

$$C_1 = C_0\, e^{-\gamma_{eff} \frac{x}{u}}$$

$$\gamma_{eff} = \frac{\alpha\gamma}{\alpha + \gamma}$$

From the data you can measure γ_{eff}

$$\gamma_{eff} \approx 0.019\ s^{-1}$$

$$\therefore 0.019 = \frac{\alpha\gamma}{\alpha+\gamma} \qquad \alpha = 0.4$$

Solving for $\gamma \Rightarrow 0.02\ s^{-1}$ slightly higher but close

(4) S.S. equations

$$u \frac{\delta c_1}{\delta x} = -\alpha c_1 + \alpha c_2 - \gamma_1 c_1$$

$$0 = \alpha c_1 - \alpha c_2 - \gamma_2 c_2$$

$$c_2 = \frac{\alpha c_1}{\alpha + \gamma_2}$$

$$\therefore \quad u \frac{\delta c_1}{dx} = -(\alpha + \gamma_1) c_1 + \frac{\alpha^2 c_1}{\alpha + \gamma_2}$$

Solving $$c_1 = c_0 e^{+\left(-\alpha - \gamma_1 + \frac{\alpha^2}{\alpha + \gamma_2}\right)\frac{x}{u}}$$

$$\gamma_{eff} = \alpha + \gamma_1 - \frac{\alpha^2}{\alpha + \gamma_2}$$

When $\gamma_2 = 0 \Rightarrow \gamma_{eff} = \gamma_1$

$\gamma_1 = 0 \Rightarrow \gamma_{eff} = \dfrac{\alpha \gamma_2}{\alpha + \gamma_2}$ consistent